TRAINING COURSES ON APPLIED ECONOMETRIC ANALYSIS (SUMMER SCHOOL) FOR YOUNG ECONOMISTS / RESEARCHERS 2016-2017

Basics of Probability theory

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Outline

- **Session 3 Sets and Basic probability concepts**
  - (Wed 14 Sept at 13:30-15:00)
- **Session 4 Probability distributions (Discrete)**
  - (Wed 14 Sept at 15:30-17:00)
- **Session 1 Probability distributions (Continuous)**
  - (Thu 15 Sept at 9:00-10:30)
- **Session 2 Inferential statistics and Confidence intervals**
  - (Thu 15 Sept at 11:00-12:30)

Session 3 Sets and Basic probability concepts

- Operations on sets (Union, Intersection and Complement)
- Concepts of experiment, outcome, event,
- Approaches to probability (Classical and Empirical)
- Relationships of events (Mutually Exclusive, Dependent and Independent, Joint probability)

Set

Set is a finite or infinite collection of objects (called elements) in which order has no significance.

Examples: $D = \{0,1,2, \dots, 9\} = \{1,2,3, \dots, 9,0\}$,

$C = \{Kazakhstan, Kyrgyzstan, Tajikistan, Turkmenistan, Uzbekistan\}$,

$N = \{1,2,3, \dots\}$.

Ways of presenting sets:

Enumeration: $A = \{x_1, x_2, \dots, x_n\}$

Description: $B = \{x | property\}$

Examples:

Enumeration: $D = \{0,1,2, \dots, 9\}$

Description: $D = \{x \in Z | 0 \leq x \leq 9\}$

Operations on sets

$\boldsymbol{A} \cup \boldsymbol{B}$ $(A \; union \; B)$ holds all elements of A or B or both.

$\boldsymbol{A} \cap \boldsymbol{B}$ $(A \; intersection \; B)$ holds the elements of both A and B.

$\boldsymbol{A}^{\boldsymbol{C}}$ $(A \; compliment)$ holds all elements that are not in A.

Example: Tossing a die. $S = \{1,2,3,4,5,6\}$.

$$E = \{2,4,6\},\ O = \{1,3,5\},\ P = \{2,3,5\}.$$

$E \cup P = \{2,3,4,5,6\}$;

$O \cap P = \{3,5\}$;

$E^C = \{1,3,5\}$;

Exercise: Find: 1) $O \cup P$; 2) $E \cap P$; 3) P^C.

Probability

- Any activity involves a chance (or probability)
- Forecasting (chance of raining or not)
- Lottery or any game (chance of winning or losing)
- Running a business (chance of succeeding or failing)
- Investing (chance of making extra money or not)
- Buying a product (chance of normal or defective)
- Taking an exam (chance of passing or failing)

Exercise: You give an example

Experiment and Sample space

Experiment is any activity or process that generates well-defined outcomes (or results).

Sample space for an experiment is the set of all experimental outcomes.

Experiment	Experimental outcomes	Sample space
Toss a coin	Head, Tail	S = {Head, Tail}
Apply for a job	Hired, Not hired	S = {Hired, Not Hired}
Roll a die	1, 2, 3, 4, 5, 6	S = {1, 2, 3, 4, 5, 6}
Play a game	Win, Lose, Draw	S = {Win, Lose, Draw}
Run a business	Profit, Loss, Even	S = {Profit, Loss, Even}

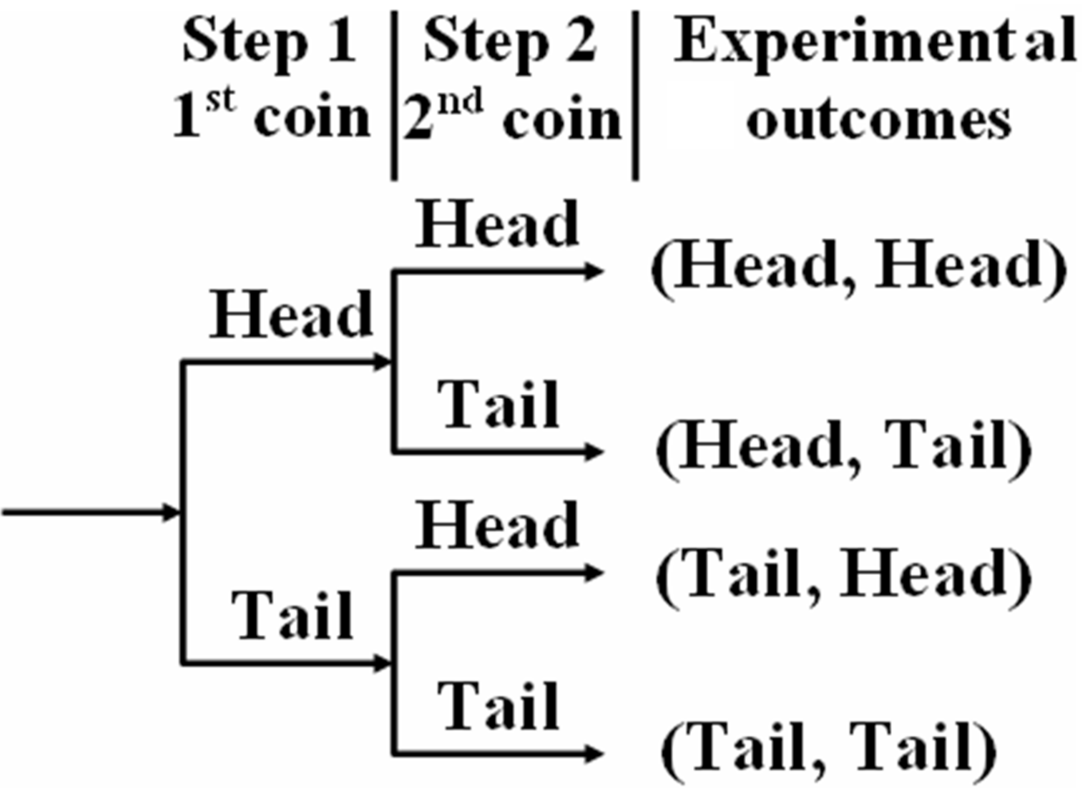
Three types of experiments

- **Multiple-step**
- **Combination**
- **Permutation**

Multiple-step experiment

- k steps with a certain number of outcomes at each step
- Total number of outcomes = $n_1 * n_2 * \ldots * n_k$.

Example: The experiment of tossing two coins.



Total number of outcomes =

$$n_1 * n_2 = 2*2 = 4$$

S = {(H, H), (H, T), (T, H), (T, T)}

Exercise:

1) How many outcomes are there if 3 coins are tossed?
2) How many outcomes are there if a coin is tossed and a die is rolled?

Combination

- ***n*** **items must be selected from a set of** ***N*** **items, where the order of selection does not matter**
- **Combinations of** ***N*** **items taken** ***n*** **at a time:**

$$C_N^n = \frac{N!}{n!\,(N-n)!}, n! = 1 \cdot 2 \cdot 3 \cdots n$$

Example: How many ways can 2 out of 5 employees (A, B, C, D, E) be chosen for a business trip?

Solution: $N = 5$ and $n = 2$. Total number of outcomes:

$$C_5^2 = \frac{5!}{2!\,(5-2)!} = \frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5}{1 \cdot 2 \cdot 1 \cdot 2 \cdot 3} = 10$$

S = {AB, AC, AD, AE, BC, BD, BE, CD, CE, DE}

Permutation

- ***n* items must be selected from a set of *N* items, where the order of selection is important**
- **Permutations of *N* items taken *n* at a time:**

$$P_N^n = \frac{N!}{(N-n)!}, \left(compare \quad C_N^n = \frac{N!}{n!\,(N-n)!} \right)$$

Example: How many ways can a director and a secretary be chosen out of 5 employees (A, B, C, D, E)?

Solution: N = 5 and n = 2. Total number of outcomes:

$$P_5^2 = \frac{5!}{(5-2)!} = \frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5}{1 \cdot 2 \cdot 3} = 20$$

S = {AB, AC, AD, AE; BA, BC, BD, BE; CA, CB, CD, CE;
DA, DB, DC, DE; EA, EB, EC, ED}

Exercises

1) An investor has two stocks: stock A and stock B. Each stock may increase in value, decrease in value, or remain unchanged. Consider the experiment of investing in the two stocks and observing the change (if any) in value. How many experimental outcomes are possible?
2) Suppose there are 8 runners in a race. How many ways can awards for 1st, 2nd, and 3rd places be presented?
3) A computer programming team has 12 members. How many ways can a group of 7 (out of the 12 members) be chosen to work on a special project?

Event

Event is a subset of the sample space. Alternatively, it is a collection (group) of outcomes.

Simple event contains a single outcome.

Example:

The experiment of rolling a die has a sample space of six outcomes:

$$S = \{1, 2, 3, 4, 5, 6\}.$$

1) Simple events: $E_1 = \{1\}$, $E_2 = \{2\}$, $E_3 = \{3\}$, $E_4 = \{4\}$, $E_5 = \{5\}$, $E_6 = \{6\}$.
2) A is an event of an even number: $A = \{2, 4, 6\}$.
3) B is the event of an odd number: $B = \{1, 3, 5\}$.

Probability

Probability is a numerical measure of the chance (or likelihood) that a particular event will occur.

Probability values are always assigned on a scale of 0 to 1, i.e.

$$0 \leq P\ (E) \leq 1.$$

A probability near 0 indicates that an event is very unlikely to occur, whereas near 1 indicates that an event is almost certain to occur.

Two approaches to probabilities

- **Classical**
- **Empirical (Frequency)**

Classical probability

$$P(event) = \frac{Number\ of\ ways\ the\ event\ can\ occur}{Total\ number\ of\ possible\ outcomes}$$

Examples:

1) For the experiment of rolling a die:

$$P(1) = \frac{Number\ of\ ways\ 1\ can\ occur}{Total\ number\ of\ possible\ outcomes} = \frac{1}{6} = 16.67\%$$

2) Santa Claus has 40 balls in his bag: 30 Red and 10 Blue. He randomly picks a ball to present to a boy.

$$P(R) = \frac{Number\ of\ ways\ R\ can\ occur}{Total\ number\ of\ possible\ outcomes} = \frac{30}{40} = 75\%$$

Empirical (frequency) Probability

Empirical probability of an event refers to the frequency of similar events happened in the past.

Example:

In 2006 after exceptionally heavy rainfall, parts of Britain experienced floods. For a total of 90 rainy days there were 2 floods. On the basis of this information, what is the probability that a future rainy day would cause floods?

$$P(Flood) = \frac{Floods}{Rainy\ days} = \frac{2}{90} = 0.022$$

Exercise

1) Tossing a die. S = {1, 2, 3, 4, 5, 6}. A = {2, 4, 6}, B = {1, 3, 5}, C = {2, 3, 5}. Find: 1) $P(A)$; 2) $P(A \cup B)$; 3) $P(A \cap C)$; 4) $P(A \cap B)$.
2) A radio station asked a random sample of 1000 out of the 250,000 listeners to find out how many preferred one of four types of music.

Type of music	Number of listeners
Popular	521
Big band	188
Classical	207
Other	84

What is the probability that a randomly chosen listener from the sample likes pop music?

The addition rule for union

The addition rule is used to compute the probability of the union of two events:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Example: Out of 200 students taking a mathematics course 160 passed the midterm exam, 140 passed the final exam and 124 students passed both exams. After reviewing the grades, the professor decides to give a passing grade to any student who passed at least one of the two exams. What is the probability of passing this course?

Let M = {pass midterm}, F = {pass final}, $M \cap F$= {pass both}, then:

$$P(M) = \frac{160}{200} = 0.80, P(F) = \frac{140}{200} = 0.70, P(M \cap F) = \frac{124}{200} = 0.62$$

According to the addition rule:

$$P(M \cup F) = P(M) + P(F) - P(M \cap F) = 0.80 + 0.70 - 0.62 = 0.88$$

Relationships of events

- Mutually exclusive events
- Dependent events
- Independent events

Mutually exclusive events

The events A and B are said to be mutually exclusive if the events do not have any experimental outcomes in common:

$$A \cap B = \emptyset \quad (P(A \cap B) = 0)$$

Examples:

1) If A = {a person is over 60 years old}, B = {a person is under 18}, then A and B are MEE.
2) Rolling a die once and getting a 2 and a 3.

Addition rule (simplified) for mutually exclusive events:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = P(A) + P(B)$$

Example: For the rolling a die, let $A = \{2\}$, $B = \{3\}$, then:

$$P(A \cup B) = P(A) + P(B) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$$

Dependent/Independent events

If an event has an influence on another event the second event is called a dependent event.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Example: Let A = {an accident}, B = {a driver has drunk}.

The events A and B are dependent, because the probability of an accident increases if the driver has drunk alcohol.

If one event has no effect on another event these events are called independent.

$$P(A|B) = P(A)$$

Example: Let A = {pass math exam}, B = {blue eyes}. The events are not influenced by each other, therefore, they are independent. $P(A|B) = P(A)$.

Multiplication rule for intersection

The multiplication rule is used to compute the probability of the intersection of two events

$$P(A \cap B) = P(B)P(A|B) \quad \left(derives\ from\ P(A|B) = \frac{P(A \cap B)}{P(B)} \right)$$

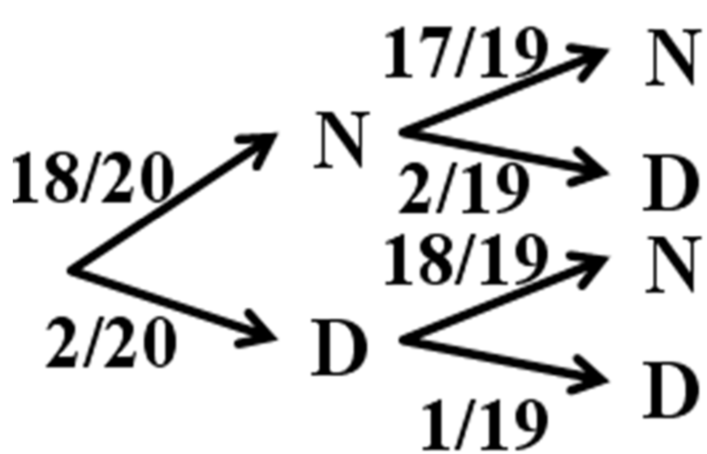
The multiplication rule (simplified) for independent events:

$$P(A \cap B) = P(B)P(A)$$

Probability tree diagram

Probability tree diagram is used to show the outcomes of an experiment and their relative probabilities

Example: 2 out of 20 products are defective at a store. If a customer randomly buys two of the products, what is the probability of both being defective?



$$P(N \cap N) = (18/20)*(17/19) = 80.53\%$$

$$P(N \cap D) = (18/20)*(2/19) = 9.47\%$$

$$P(D \cap N) = (2/20)*(18/19) = 9.47\%$$

$$P(D \cap D) = (2/20)*(1/19) = 0.53\%$$

Exercise: What is the probability of at least one of them being defective?

Joint probability table

Contingency table is a table of joint-occurrence frequencies of two variables.

Joint probability table is a table with probabilities calculated based on the contingency table.

Example: The table shows the questionnaire results of 100 employees of a company

	Happy	**Unhappy**	**Total**
Men	30	10	40
Women	30	30	60
Total	60	40	100

$\Rightarrow$

	Happy	**Unhappy**	**Total**
Men	0.3	0.1	0.4
Women	0.3	0.3	0.6
Total	0.6	0.4	1

Find the probabilities that: **a)** a man is unhappy; **b)** an employee is a man and unhappy; **c)** an employee is a woman and unhappy.

Thank you