

A review of propensity score: principles, methods and application in Stata

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Theoretical background

Application in Stata

The central role of the propensity score in observational studies for causal effects

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SUMMARY

The propensity score is the conditional probability of assignment to a particular treatment given a vector of observed covariates. Both large and small sample theory show that adjustment for the scalar propensity score is sufficient to remove bias due to all observed covariates. Applications include: (i) matched sampling on the univariate propensity score, which is a generalization of discriminant matching, (ii) multivariate adjustment by subclassification on the propensity score where the same subclasses are used to estimate treatment effects for all outcome variables and in all subpopulations, and (iii) visual representation of multivariate covariance adjustment by a two-dimensional plot.

Causal inference framework

ID	T	Y
1	0	21
2	1	31
...
n	1	15

$$T \rightarrow Y$$

Potential outcomes

ID	T	Y	Y(t=0)	Y(t=1)
1	0	21	21	22
2	1	31	16	31
...
n	1	15	15	15

$$T \rightarrow Y$$

Potential outcomes

ID	T	Y	Y(0)	Y(1)
1	0	21	21	22
2	1	31	16	31
...
n	1	15	15	15

$$T \rightarrow Y$$

Potential outcomes

ID	T	Y	Y(0)	Y(1)
1	0	21	21	22
2	1	31	16	31
...
n	1	15	15	15

$$T \rightarrow Y$$

Potential outcomes

ID	T	Y(0)	Y(1)
1	0	21	22
2	1	16	31
...
n	1	15	15

$$T \rightarrow Y$$

Individual treatment effect

ID	T	Y(0)	Y(1)
1	0	21	22
2	1	16	31
...
n	1	15	15

$$\tau_i = Y_i(1) - Y_i(0)$$

Fundamental problem of causal inference

ID	T	Y(0)	Y(1)
1	0	21	.
2	1	.	31
...
n	1	.	15

$$\tau_i = Y_i(1) - Y_i(0)$$

Average treatment effect (ATE)

ID	T	Y(0)	Y(1)
1	0	21	.
2	1	.	31
...
n	1	.	15

$$ATE = E[Y(1) - Y(0)] = E[Y(1)] - E[Y(0)]$$

Average treatment effect among treated (ATT)

ID	T	Y(0)	Y(1)
1	0	21	.
2	1	.	31
...
n	1	.	15

$$ATT = E[Y(1) - Y(0) / T = 1] = E[Y(1) / T = 1] - E[Y(0) / T = 1]$$

Potential outcome means

ID	T	Y(0)	Y(1)
1	0	21	.
2	1	.	31
...
n	1	.	15

$$ATE = E[Y(1)] - E[Y(0)]$$

$$ATT = E[Y(1)/T = 1] - E[Y(0)/T = 1]$$

Potential outcome means

ID	T	Y(0)	Y(1)
1	0	21	.
2	1	.	31
...
n	1	.	15

$$(Y(1), Y(0)) \perp T$$

Potential outcome means

ID	T	Y(0)	Y(1)	X_1	X_2	...	X_p
1	0	21	.				
2	1	.	31				
...				
n	1	.	15				

$$(Y(1), Y(0)) \not\perp T$$

Potential outcome means

ID	T	Y(0)	Y(1)	X
1	0	21	.	
2	1	.	31	
...	
n	1	.	15	

$$(Y(1), Y(0)) \not\perp T$$

Strong ignorability assumption

ID	T	Y(0)	Y(1)	X
1	0	21	.	
2	31	.	1	
...	
n	1	.	15	

$$(Y(1), Y(0)) \perp T | \mathbf{X}$$

$$0 < P(T = 1 | \mathbf{X}) < 1$$

Adjusting for X

- ▶ Regression
- ▶ Matching
- ▶ Stratification

If...

ID	T	Y(0)	Y(1)	X
1	0	21	.	
2	1	.	31	
...	
n	1	.	15	

$$(Y(1), Y(0)) \perp T | \mathbf{X}$$

$$0 < P(T = 1 | \mathbf{X}) < 1$$

Then...

ID	T	Y(0)	Y(1)	X	b(X)
1	0	21	.		
2	1	.	31		
...		
n	1	.	15		

$$(Y(1), Y(0)) \perp T | b(\mathbf{X})$$

$$0 < P(T = 1 | b(\mathbf{X})) < 1$$

Balancing score

ID	T	Y(0)	Y(1)	X	b(X)
1	0	21	.		
2	1	.	31		
...		
n	1	.	15		

$$\mathbf{X} \perp T | b(\mathbf{X})$$

Propensity score

ID	T	Y(0)	Y(1)	X	b(X)	e(X)
1	0	21	.			
2	1	.	31			
...			
n	1	.	15			

$$e(\mathbf{X}) = P(T = 1|\mathbf{X})$$

$$\mathbf{X} \perp T | e(\mathbf{X})$$

Propensity score

ID	T	Y(0)	Y(1)	X	b(X)	e(X)
1	0	21	.			
2	1	.	31			
...			
n	1	.	15			

$$e(\mathbf{X}) = P(T = 1|\mathbf{X})$$

Propensity score is the coarsest balancing score: $e(\mathbf{X})=f(b(\mathbf{X}))$

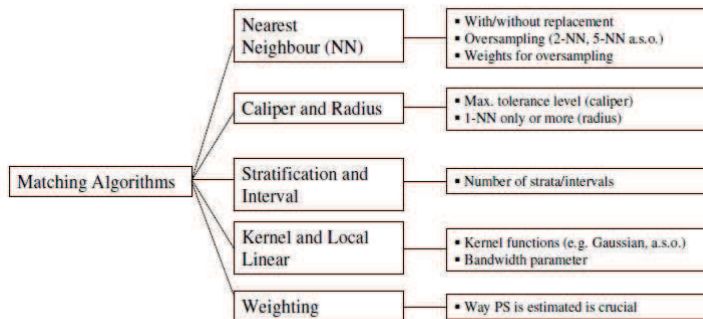
Adjusting for $e(X)$

- ▶ Matching
- ▶ Stratification
- ▶ Regression

Matching

- ▶ most popular propensity score based method
- ▶ we match subjects from the treatment groups by $e(\mathbf{X})$
- ▶ subjects who are unable to be matched are discarded from the analysis

Different matching algorithms have been proposed



Some practical guidance for the implementation of propensity score matching (Caliendo, 2005)

Nearest neighbor matching

$$ATT = \frac{1}{N^T} \sum_{i \in T} [Y_i^T - \sum_{j \in C(i)} w_{ij} Y_j^C]$$

- ▶ N^T number of treated units
- ▶ $C(i)$ set of controls matched to treated unit i
- ▶ N_i^C number of controls matched to treated unit i
- ▶ $w_{ij} = \frac{1}{N_i^C}$ if $j \in C(i)$; 0, otherwise

Stratification

- ▶ using $e(\mathbf{x})$, we stratify the entire sample into quantiles
- ▶ within each stratum, we assess the treatment effect
- ▶ we compute an overall treatment effect by averaging the results for each stratum

Regression

- ▶ $e(\mathbf{x})$ is included in the **outcome** regression model
- ▶ **with/without** other covariates
- ▶ we assume a **linear** relationship between $e(\mathbf{x})$ and Y

Estimation of propensity score

We can estimate propensity score using logistic regression

$$P(T = 1 \mid X_1, \dots, X_p) = \frac{\exp(\beta_0 + \beta_1 X_1 + \dots + \beta_p X_p)}{1 + \exp(\beta_0 + \beta_1 X_1 + \dots + \beta_p X_p)}$$

Estimation of propensity score

Adequacy of propensity score model can be checked evaluating the balance of X across treatment groups.

$$\mathbf{X} \perp T | \widehat{e(\mathbf{X})}$$

- ▶ Statistic tests
- ▶ Standardized differences
- ▶ Graphical methods

Theoretical background

Application in Stata

Does homelessness affect physical health?

- ▶ **treatment**: homelessness
- ▶ **outcome**: physical health, measured by the Physical Component Summary (PCS) score
- ▶ **confounders**: age, sex, alcohol intake, Mental Component Summary (MCS) score

HELP (Health Evaluation and Linkage to Primary Care) dataset

- ▶ PSCORE (Becker and Ichino, 2002)
- ▶ estimation of propensity score

PSCORE - syntax

```
. ssc install pscore  
. help pscore  
  
. pscore exposure varlist  
  
. pscore homeless age gender alc mcs, ///  
logit pscore(mypscore)
```

PSCORE - output

The treatment is homeless

homeless	Freq.	Percent	Cum.
0	244	53.86	53.86
1	209	46.14	100.00
Total	453	100.00	

Logistic regression

Number of obs = 453
LR chi2(4) = 32.75
Prob > chi2 = 0.0000
Pseudo R2 = 0.0524

Log likelihood = -296.26879

homeless	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
age	.013659	.0129651	1.05	0.292	-.0117521	.0390701
female	-.4545305	.2370108	-1.92	0.055	-.9190632	.0100022
alc	.024878	.0057815	4.30	0.000	.0135464	.0362096
mcs	-.0099835	.0077682	-1.29	0.199	-.0252089	.005242
_cons	-.6582588	.518953	-1.27	0.205	-1.675388	.3588703

PSCORE tests the **balancing hypothesis** through this algorithm:

1. Split the sample in k equally spaced intervals of $e(\mathbf{x})$
2. Within each interval test that the average $e(\mathbf{x})$ of treated and untreated do not differ

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1. Split the sample in k equally spaced intervals of $e(\mathbf{x})$
2. Within each interval test that the average $e(\mathbf{x})$ of treated and untreated do not differ
3. If the test fails, split the interval and test again
4. Continue until, in all intervals, the average $e(\mathbf{x})$ of treated and untreated units do not differ

PSCORE - balance checking

Test for block 3

Two-sample t test with equal variances

Group	Obs	Mean	Std. Err.	Std. Dev.	[95% Conf. Interval]	
0	105	.4751954	.0052945	.0542521	.4646963	.4856945
1	113	.4848887	.0054481	.0579139	.4740941	.4956834
combined	218	.4802199	.0038105	.0562608	.4727097	.4877302
diff		-.0096933	.0076152		-.0247029	.0053163
diff = mean(0) - mean(1)				t = -1.2729		
Ho: diff = 0				degrees of freedom = 216		
Ha: diff < 0		Ha: diff != 0		Ha: diff > 0		
Pr(T < t) = 0.1022		Pr(T > t) = 0.2044		Pr(T > t) = 0.8978		

PSCORE tests the **balancing hypothesis** through this algorithm:

1. Split the sample in k equally spaced intervals of $e(\mathbf{x})$
2. Within each interval test that the average $e(\mathbf{x})$ of treated and untreated do not differ
3. If the test fails, split the interval and test again
4. Continue until, in all intervals, the average $e(\mathbf{x})$ of treated and untreated units do not differ
5. Within each interval, test that the means of each characteristic do not differ between treated and untreated

PSCORE - balance checking

Testing the balancing property for variable age in block 3

Two-sample t test with equal variances

Group	Obs	Mean	Std. Err.	Std. Dev.	[95% Conf. Interval]	
0	105	36.22857	.7236529	7.415235	34.79354	37.6636
1	113	36.47788	.7777827	8.267943	34.9368	38.01895
combined	218	36.3578	.5317885	7.851764	35.30967	37.40593
diff		-.2493047	1.06662		-2.35162	1.853011
diff = mean(0) - mean(1)				t = -0.2337		
Ho: diff = 0				degrees of freedom = 216		
Ha: diff < 0		Ha: diff != 0		Ha: diff > 0		
Pr(T < t) = 0.4077		Pr(T > t) = 0.8154		Pr(T > t) = 0.5923		

PSCORE - balance checking

Testing the balancing property for variable age in block 3

Two-sample t test with equal variances

Group	Obs	Mean	Std. Err.	Std. Dev.	[95% Conf. Interval]	
0	105	36.22857	.7236529	7.415235	34.79354	37.6636
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diff		-.2493047	1.06662		-2.35162	1.853011
diff = mean(0) - mean(1)				t = -0.2337		
Ho: diff = 0				degrees of freedom = 216		
Ha: diff < 0		Ha: diff != 0		Ha: diff > 0		
Pr(T < t) = 0.4077		Pr(T > t) = 0.8154		Pr(T > t) = 0.5923		

The balancing property is satisfied

Matching on $e(X)$

- ▶ PSMATCH2 (Leuven and Sianesi, 2003)
- ▶ propensity score matching

PSMATCH2 - syntax

```
. ssc install psmatch2
. help psmatch2

. psmatch2 depvar [indvar], ///
outcome(varname) pscore(varname)

. psmatch2 homeless, ///
outcome(pcs) pscore(mypscore) neighbor(1)

. psmatch2 homeless age gender alc mcs, ///
outcome(pcs) neighbor(1)
```

PSMATCH2 - output

```
. psmatch2 homeless, outcome(pcs) pscore(mypscore) neighbor(1)
```

Variable	Sample	Treated	Controls	Difference	S.E.	T-stat
pcs	Unmatched	46.9367801	49.000829	-2.06404896	1.0129221	-2.04
	ATT	46.9367801	47.8530158	-.916235746	1.48207493	-0.62

Note: S.E. does not take into account that the propensity score is estimated.

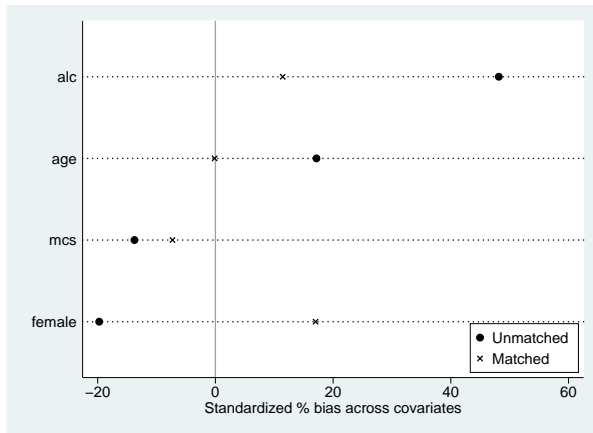
PSTEST assesses balance in the matched samples

```
. pstest age female alc mcs, both graph
```

Variable	Unmatched Matched	Mean		%bias	%reduct bias	t-test		V(T) / V(C)
		Treated	Control			t	p> t	
age	U	36.368	35.041	17.2		1.83	0.068	1.33*
	M	36.368	36.378	-0.1	99.3	-0.01	0.990	1.24
female	U	.19139	.27459	-19.7		-2.08	0.038	.
	M	.19139	.11962	17.0	13.7	2.03	0.043	.
alc	U	23.038	13.512	48.1		5.19	0.000	2.38*
	M	23.038	20.775	11.4	76.2	1.06	0.291	1.35*
mcs	U	30.731	32.487	-13.7		-1.45	0.147	0.81
	M	30.731	31.663	-7.3	46.9	-0.78	0.435	0.97

* if variance ratio outside [0.76; 1.31] for U and [0.76; 1.31] for M

PSTEST - output



PSMATCH2 - output

```
. psmatch2 homeless, outcome(pcs) pscore(mypscore) radius caliper(0.03)
```

Variable	Sample	Treated	Controls	Difference	S.E.	T-stat
pcs	Unmatched	46.9367801	49.000829	-2.06404896	1.0129221	-2.04
	ATT	47.172174	48.4370461	-1.26487214	1.07716821	-1.17

Note: S.E. does not take into account that the propensity score is estimated.

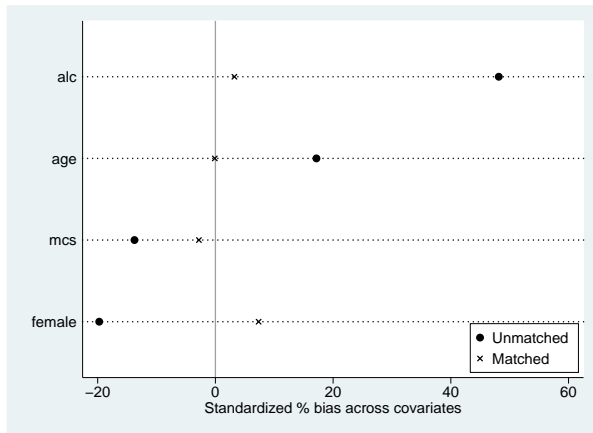
PSTEST assesses balance in the matched samples

```
. pstest age female alc mcs, both graph
```

Variable	Unmatched Matched	Mean		%bias	%reduct bias	t-test		V(T)/ V(C)
		Treated	Control			t	p> t	
age	U	36.368	35.041	17.2		1.83	0.068	1.33*
	M	36.167	36.175	-0.1	99.4	-0.01	0.992	1.25
female	U	.19139	.27459	-19.7		-2.08	0.038	.
	M	.19704	.16609	7.3	62.8	0.81	0.420	.
alc	U	23.038	13.512	48.1		5.19	0.000	2.38*
	M	20.443	19.804	3.2	93.3	0.36	0.716	1.04
mcs	U	30.731	32.487	-13.7		-1.45	0.147	0.81
	M	30.772	31.129	-2.8	79.7	-0.29	0.772	0.88

* if variance ratio outside [0.76; 1.31] for U and [0.76; 1.32] for M

PSTEST - output



PSMATCH2 - output

```
. gen femmcs=female*mcs
. gen femalc=female*alc
. gen alcmcs=alc*mcs

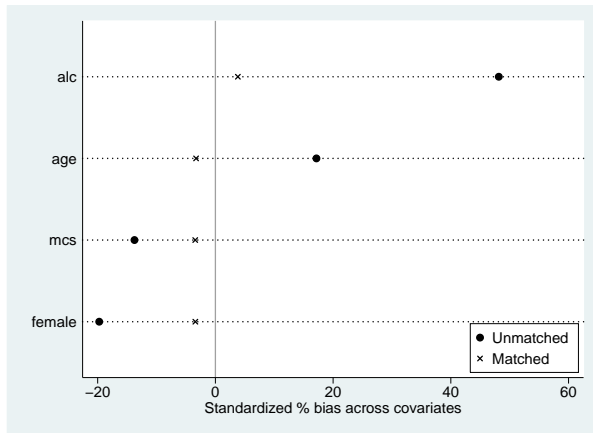
. pscore homeless age female alc mcs femmcs femalc alcmcs, logit pscore(mypscore)

. psmatch2 homeless, outcome(pcs) pscore(mypscore) neighbor(1)
```

Variable		Sample	Treated	Controls	Difference	S.E.	T-stat
pcs	Unmatched		46.9367801	49.000829	-2.06404896	1.0129221	-2.04
	ATT		46.9367801	47.5441296	-.607349496	1.41574607	-0.43

Note: S.E. does not take into account that the propensity score is estimated.

PSTEST - output



PSMATCH2 - output

```
. pscore homeless age female alc mcs femmcs femalc alcmcs, logit pscore(mypscore)
```

```
. psmatch2 homeless, outcome(pcs) pscore(mypscore) neighbor(1)
```

Variable		Sample	Treated	Controls	Difference	S.E.	T-stat
pcs	Unmatched		46.9367801	49.000829	-2.06404896	1.0129221	-2.04
	ATT		46.9367801	47.5441296	-.607349496	1.41574607	-0.43

Note: S.E. does not take into account that the propensity score is estimated.

Standard error:

- ▶ Lechner (2001)
- ▶ Abadie et al. (2004)
- ▶ Abadie and Imbens (2006)

PSMATCH2 - output

```
. pscore homeless age female alc mcs femmcs femalc alcmcs, logit pscore(mypscore)
```

```
. psmatch2 homeless, outcome(pcs) pscore(mypscore) neighbor(1)
```

Variable		Sample	Treated	Controls	Difference	S.E.	T-stat
pcs	Unmatched		46.9367801	49.000829	-2.06404896	1.0129221	-2.04
	ATT		46.9367801	47.5441296	-.607349496	1.41574607	-0.43

Note: S.E. does not take into account that the propensity score is estimated.

Source of variability:

- ▶ propensity score estimation
- ▶ matching on the common support
- ▶ order in which treated individuals are matched

Set of commands to estimate ATE and ATT (ATET) through:

- ▶ regression adjustment
- ▶ inverse probability weighting
- ▶ PS matching
- ▶ ...

TEFFECTS - syntax

```
. teffect psmatch (outcome variable) ///  
(treat treatvarlist [, treatmodel])
```

```
. teffects psmatch (pcs)    ///  
(homeless age female alc mcs  ///  
femalc femmcs alcmcs, logit), atet
```


TEFFECTS - output

```
. teffects psmatch (pcs) (homeless age female alc mcs femalc femmcs alcmcs, logit), atet
```

```
Treatment-effects estimation      Number of obs      =      453
Estimator      : propensity-score matching      Matches: requested =      1
Outcome model  : matching                      min =      1
Treatment model: logit                      max =      1
```

pcs		Coef.	AI Robust Std. Err.	z	P> z	[95% Conf. Interval]	
ATET							
homeless							
(1 vs 0)		-.6073495	1.335721	-0.45	0.649	-3.225315	2.010616

Standard error:

- Abadie and Imbens (2012)

TEFFECTS vs. PSMATCH2

```
. teffects psmatch (pcs) (homeless age female alc mcs femalc femmcs alcmcs, logit), atet
```

```
Treatment-effects estimation      Number of obs      =      453
Estimator      : propensity-score matching      Matches: requested =      1
Outcome model  : matching                      min =      1
Treatment model: logit                      max =      1
```

		AI Robust		z	P> z	[95% Conf. Interval]	
pcs		Coef.	Std. Err.				
ATET							
homeless							
(1 vs 0)		-.6073495	1.335721	-0.45	0.649	-3.225315	2.010616

```
. psmatch2 homeless, outcome(pcs) pscore(mypscore) neighbor(1)
```

Variable		Sample	Treated	Controls	Difference	S.E.	T-stat
pcs	Unmatched		46.9367801	49.000829	-2.06404896	1.0129221	-2.04
	ATT		46.9367801	47.5441296	-.607349496	1.41574607	-0.43

Note: S.E. does not take into account that the propensity score is estimated.

- ▶ ATT* (Becker and Ichino, 2002)
- ▶ PS **matching**: ATTND, ATTNW (nearest neighbor) ATTR (radius), ATTK (kernel)
- ▶ PS **stratification**: ATTS

ATT* - syntax

```
. att* depvar [indvar], ///  
outcome(varname) pscore(varname)  
  
. attnw pcs homeless, pscore(mypscore)
```

ATT* - output

```
. attnw pcs homeless, pscore(mypscore)
```

ATT estimation with Nearest Neighbor Matching method
(equal weights version)
Analytical standard errors

n. treat.	n. contr.	ATT	Std. Err.	t
209	116	-0.607	1.416	-0.429

Note: the numbers of treated and controls refer to actual nearest neighbor matches

Sensitivity analyses

- ▶ SENSATT (Nannicini, 2007)
- ▶ after ATT*
- ▶ assesses the robustness of ATT with respect to unmeasured confounding

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