WESTMINSTER INTERNATIONAL UNIVERSITY IN TASHKENT
An Accredited Institution of the University of Westminster (UK)

INTERNATIONAL FOOD POLICY RESEARCH INSTITUTE
IFPRI

Introduction to Statistical Inference

Regional Training on Applied Econometric Analysis for Young Economists/Researchers

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Topics

- Sampling Distribution
- Hypothesis testing: one sample
- Hypothesis testing: two samples
- Completely Randomized Design: single factor (ANOVA F-test)

Sampling distribution

Suppose that we want to select a sample of *n* objects from a population of *N* objects. A **random sample** is selected such that every object has an equal probability of being selected and the objects are selected independently – the selection of one object does not change the probability of selecting any other objects.

Consider a random sample selected from a population that is used to make an inference about some population characteristic, such as the population mean, μ, using a sample statistic, such as the sample mean, $\bar{x}$. The inference is based on the realisation that every random sample has a different number for $\bar{x}$, and thus, $\bar{x}$ is a random variable. The **sampling distribution** of the sample mean is the probability distribution of the sample means obtained from <u>*all possible samples*</u> of the same number of observations drawn from the population.

Sampling distribution

Six employees, each of whose experience are

2 4 6 6 7 8

$$\mu = \frac{2+4+6+6+7+8}{6} = 5.5$$

n = 2

Sample	Sample mean	Sample	Sample mean
2, 4	3.0	4, 8	6.0
2, 6	4.0	6, 6	6.0
2, 6	4.0	6, 7	6.5
2, 7	4.5	6, 8	7.0
2, 8	5.0	6, 7	6.5
4, 6	5.0	6, 8	7.0
4, 6	5.0	7, 8	7.5
4, 7	5.5		

Sampling distribution

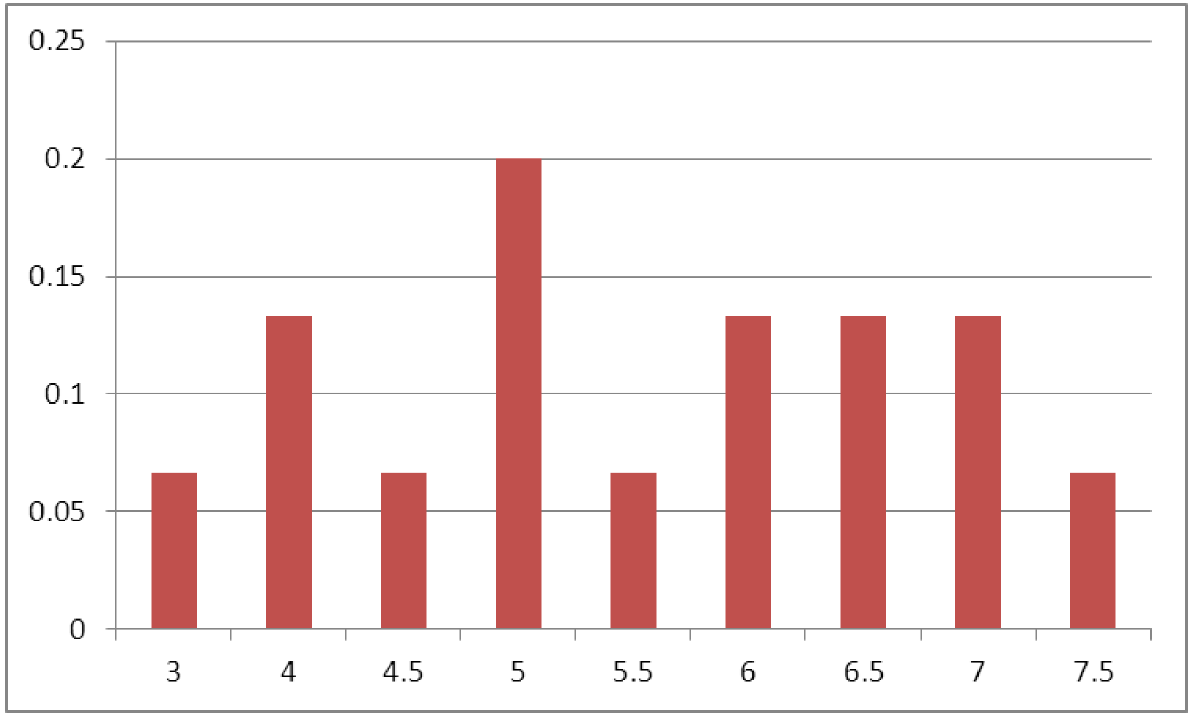
2 4 6 6 7 8

n = 2

Sample mean $\bar{x}$	Probability of $\bar{x}$
3	1/15
4	2/15
4.5	1/15
5	3/15
5.5	1/15
6	2/15
6.5	2/15
7	2/15
7.5	1/15

This table is the sampling distribution of the sample mean.

Sampling distribution

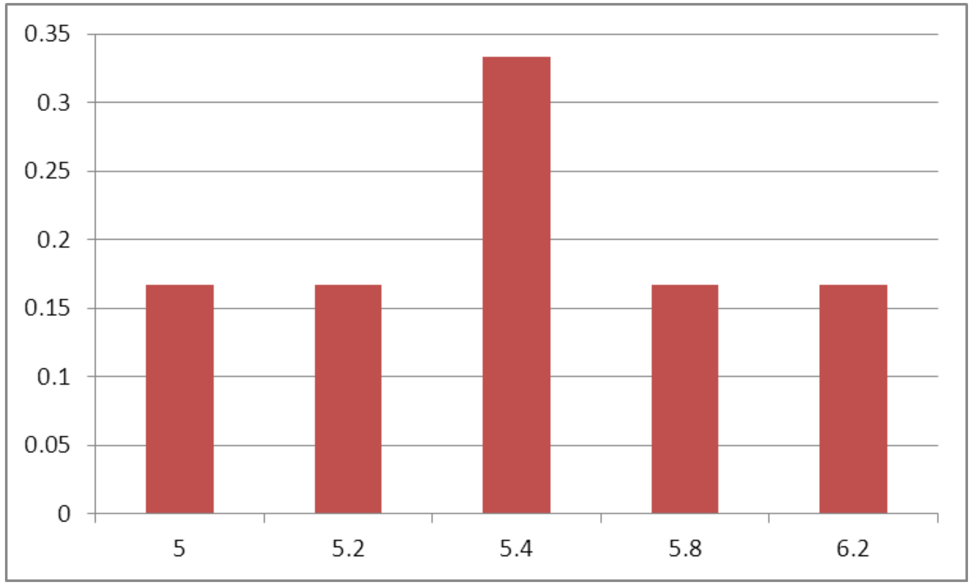


Sampling distribution

2 4 6 6 7 8

n = 5

Sample	$\overline{x}$	Probability
2, 4, 6, 6, 7	5	1/6
2, 4, 6, 6, 8	5.2	1/6
2, 4, 6, 7, 8	5.4	2/6
2, 6, 6, 7, 8	5.8	1/6
4, 6, 6, 7, 8	6.2	1/6



Sampling distribution

Let the random variables $X_1, X_2, \ldots, X_m$ denote a random sample from a population.

$\bar{X}$ is a random variable values of which are given by sample means $\bar{x}$ of random samples $X_1, X_2, \ldots, X_m$.

The mean of the sampling distribution of the sample means is the population mean.

$$E(\bar{X}) = \mu$$

Sampling distribution

n = 5

Sample	$\overline{x}$	Probability
2, 4, 6, 6, 7	5	1/6
2, 4, 6, 6, 8	5.2	1/6
2, 4, 6, 7, 8	5.4	2/6
2, 6, 6, 7, 8	5.8	1/6
4, 6, 6, 7, 8	6.2	1/6

$$E\left(\overline{X}\right) = \frac{5 + 5.2 + 5.4 + 5.4 + 5.8 + 6.2}{6} = 5.5 = \mu$$

Sampling distribution

Let the random variables $X_1, X_2, \ldots, X_n$ denote a random sample from a population.

$\bar{X}$ is a random variable values of which are given by sample means $\bar{x}$ of random samples $X_1, X_2, \ldots, X_n$.

The mean of the sampling distribution of the sample means is the population mean. $E(\bar{X}) = \mu$

The corresponding standard deviation, called **st**andard **e**rror of the **m**ean $\bar{X}$ (STEM), is given by the following:

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

These features of the sampling distribution hold as long as samples of *n* random and independent observations are repeatedly and independently drawn from a population, and the number of samples become very large.

Sampling distribution

If the population has a normal distribution, the sampling distribution of the sample means also has a normal distribution.

Central Limit Theorem

Regardless of the population distribution, as *n* becomes large $(n \geq 30)$, the sampling distribution of the sample means approaches a normal distribution.

Law of Large Numbers

Regardless of the population distribution, as *n* becomes large $(n \geq 30)$, the sample mean will approach the population mean.

When the sample size *n* is a small proportion of the population size *N*, the expected value of the sample variance is equal to the population variance:

$$E(s^2) = \sigma^2$$

Hypothesis testing: theory

Null hypothesis (H_0): a hypothesis about the specific values of one or more population parameters. The hypothesis generally represents the status quo, which we adopt until it is proven false. The hypothesis is always stated as

H_0: parameter=value

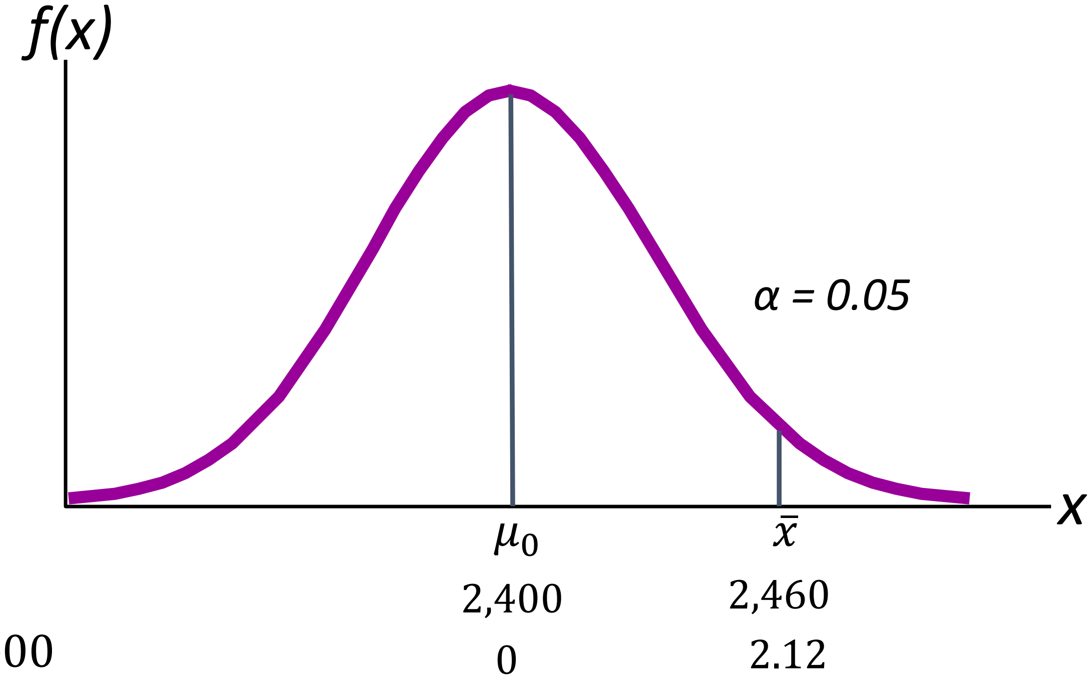
Alternative hypothesis (H_a): a hypothesis that contradicts the null hypothesis. The hypothesis generally represents that which we will adopt only when sufficient evidence exists to establish its truth.

Hypothesis testing: one-tail test

Example 1

Suppose building specifications in a certain city require that the average breaking strength of residential sewer pipe be more than 2,400 pounds per foot of length. Each manufacturer who wants to sell pipe in this city must demonstrate that its product meets the specification.

Hypothesis testing: one-tail test



$H_0\colon \mu = 2{,}400$

$H_a\colon \mu > 2{,}400$

$\bar{x} = 2{,}460$

$s = 200$

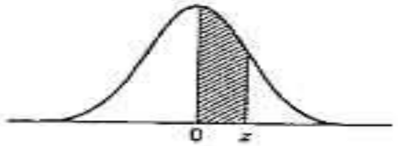
$n = 50$

$$z = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{2{,}460 - 2{,}400}{200/\sqrt{50}} = \frac{60}{28.28} = 2.12$$

$z = 2.12$

AREAS UNDER THE NORMAL CURVE

An entry in the table is the proportion under the entire curve which is between $z = 0$ and a positive value of z. Areas for negative values of z are obtained by symmetry.



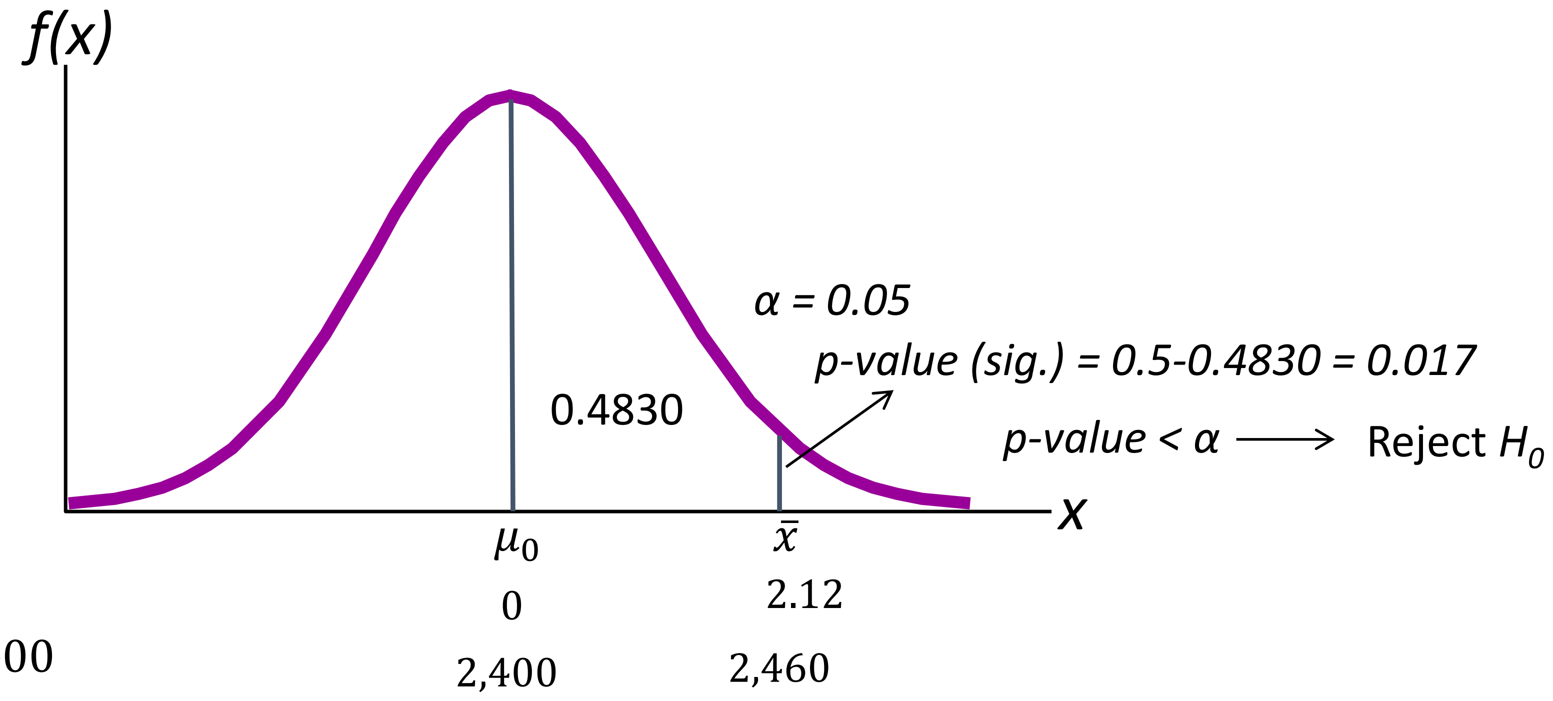
0 z

Second decimal place of z

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2703	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

From Paul G. Hoel, *Elementary Statistics*, 3rd ed., © 1971, John Wiley and Sons, Inc., New York, p. 287.

Hypothesis testing: one-tail test



H_0: $\mu = 2{,}400$

H_a: $\mu > 2{,}400$

$\bar{x} = 2{,}460$

$s = 200$

$n = 50$

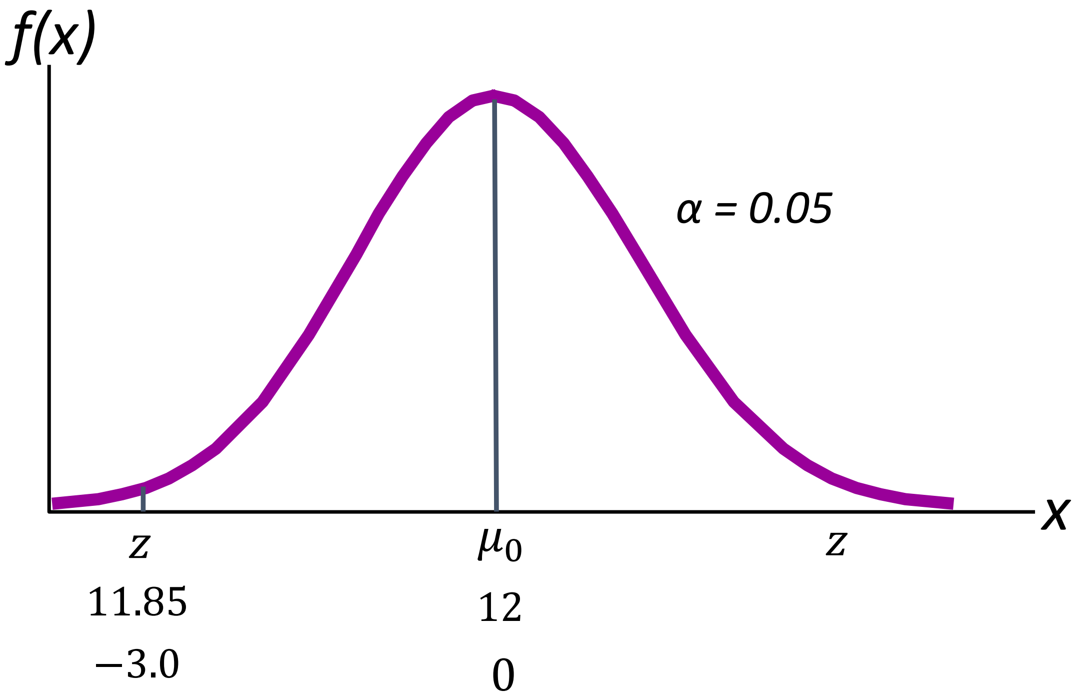
$$z = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{2{,}460 - 2{,}400}{200/\sqrt{50}} = \frac{60}{28.28} = 2.12$$

Hypothesis testing: two-tail test

Example 2

A manufacturer of cereal wants to test the performance of one of its filling machines. The machine is designed to discharge a mean amount of 12 ounces per box, and the manufacturer wants to detect any departure from this setting. This quality study calls for randomly sampling 100 boxes from today's production run and determining whether the mean fill for the run is 12 ounces per box.

Hypothesis testing: two-tail test



H_0: $\mu = 12$

H_a: $\mu \neq 12$

$\bar{x} = 11.85$

$s = 0.5$

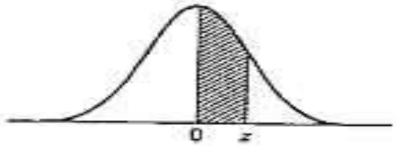
$n = 100$

$$z = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{11.85 - 12}{0.5/\sqrt{100}} = \frac{-0.15}{0.05} = -3.0$$

$z = 2.12$

AREAS UNDER THE NORMAL CURVE

An entry in the table is the proportion under the entire curve which is between $z = 0$ and a positive value of z. Areas for negative values of z are obtained by symmetry.



Second decimal place of z

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2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
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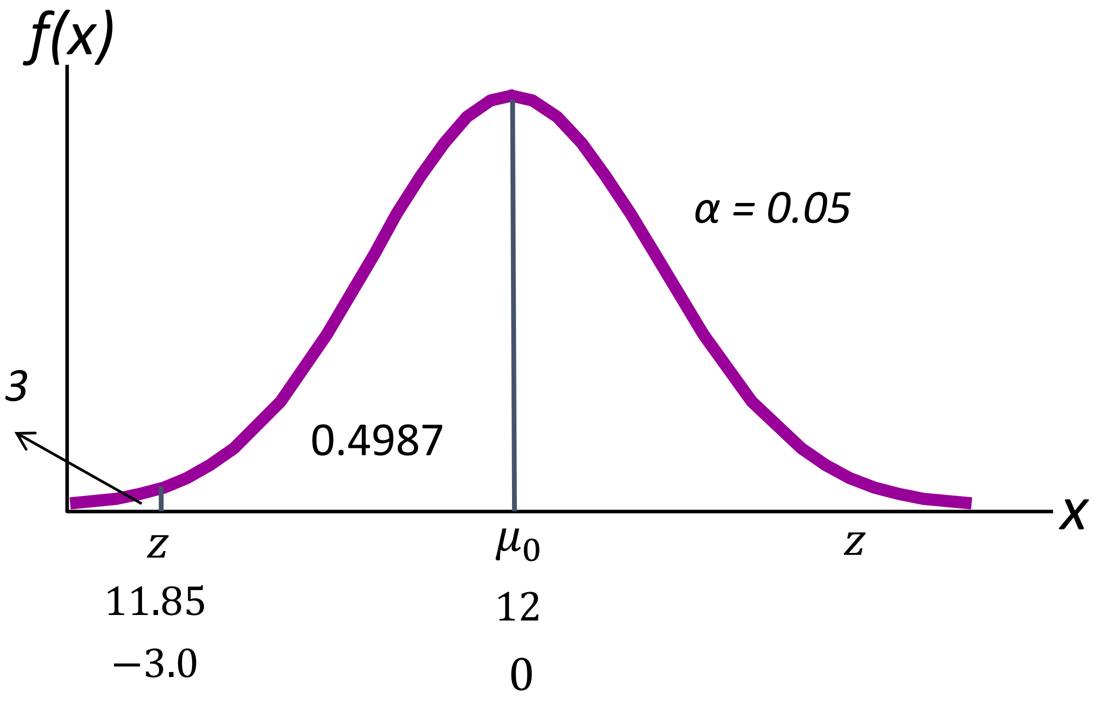
Hypothesis testing: two-tail test

p-value (sig.) = 0.5-0.4987 = 0.0013

p-value $< \alpha \longrightarrow$ Reject H_0



H_0: $\mu = 12$

H_a: $\mu \neq 12$

$\bar{x} = 11.85$

$s = 0.5$

$n = 100$

$$z = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{11.85 - 12}{0.5/\sqrt{100}} = \frac{-0.15}{0.05} = -3.0$$

Hypothesis testing - one sample: practice

Refer to the Journal of Statistics Education data on diamond saved in the DIAMOND file. The file contains data of a random sample of 308 diamonds found the mean and standard deviation of the number of carats per diamond for the sample. Let μ represent the mean number of carats in the population. Suppose you want to test $H_0: \mu = 0.6$ against $H_a:\ \mu > 0.6$.

- Use a statistical software package Stata to find the *p*-value of the test.
- Compare the *p*-value to $\alpha = 0.05$ and make the appropriate conclusion
- Compare the *p*-value to $\alpha = 0.10$ and make the appropriate conclusion.
- What do the results suggest about the choice of α in a test of hypothesis?

Independent samples

Example

In early 2000s, the United States and Japan have engaged in intense negotiations regarding restrictions on trade between two countries. One of the claims made repeatedly by US officials is that many Japanese manufacturers price their goods higher in Japan than in the United States, in effect subsidizing low prices in the United States by extremely high prices in Japan. You have to test whether this claim is true.

Independent samples

Independent random samples of retail sales in the United States and in Japan over the same time period and for the same model of automobile. The Japanese sales prices were converted from yen to dollars using current conversion rates.

<u>United States: sample 1</u>

$n_1 = 50$

$\overline{x}_1 = 16{,}596$

$s_1 = 1{,}981$

<u>Japan: sample 2</u>

$n_2 = 30$

$\overline{x}_2 = 17{,}250$

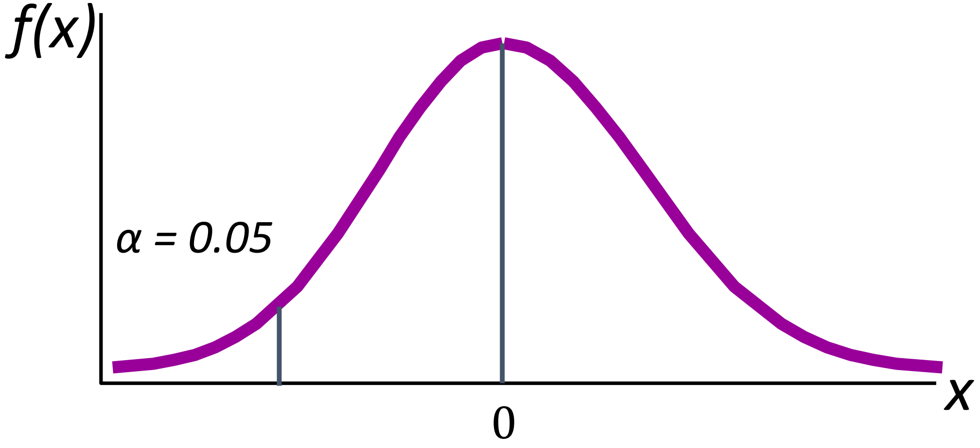
$s_2 = 1{,}865$

Hypotheses

$H_0: (\mu_1 - \mu_2) = 0$

$H_a: (\mu_1 - \mu_2) < 0$

Independent samples



$H_0: (\mu_1 - \mu_2) = 0$

$H_a: (\mu_1 - \mu_2) < 0$

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sigma_{(\bar{x}_1 - \bar{x}_2)}} = \frac{(16{,}596 - 17{,}250) - 0}{\sqrt{\frac{(1{,}981)^2}{50} + \frac{(1{,}865)^2}{30}}} = \frac{-654}{440.94} = -1.48$$

$$\sigma_{(\bar{x}_1 - \bar{x}_2)} = \sqrt{\frac{{\sigma_1}^2}{n_1} + \frac{{\sigma_2}^2}{n_2}} \approx \sqrt{\frac{{s_1}^2}{n_1} + \frac{{s_2}^2}{n_2}}$$

$z = -1.48$

AREAS UNDER THE NORMAL CURVE

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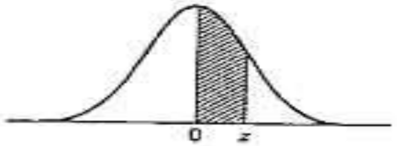

Second decimal place of z

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
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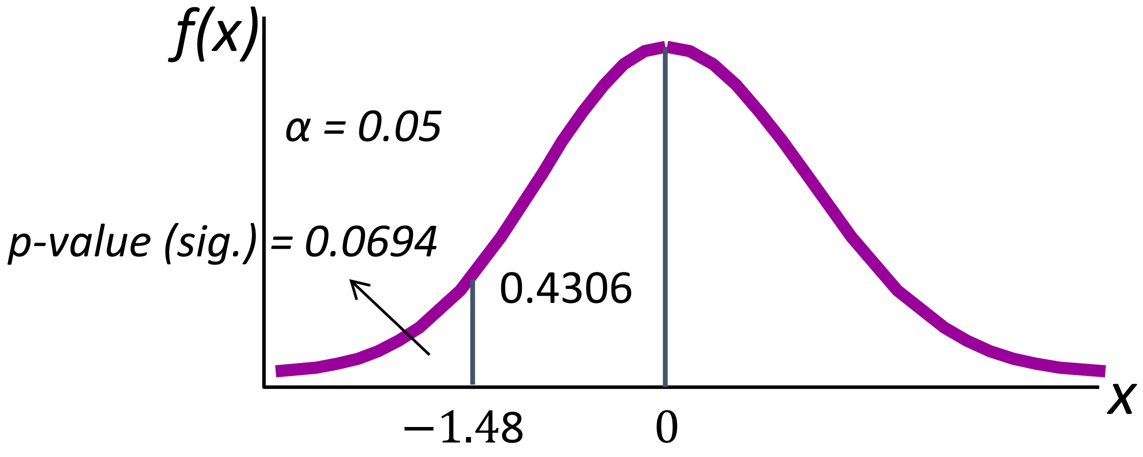
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

From Paul G. Hoel, *Elementary Statistics*, 3rd ed., © 1971, John Wiley and Sons, Inc., New York, p. 287.

Independent samples



p-value > *α* ⟶ Don't reject H_0

$H_0: (\mu_1 - \mu_2) = 0$

$H_a: (\mu_1 - \mu_2) < 0$

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sigma_{(\bar{x}_1 - \bar{x}_2)}} = \frac{(16{,}596 - 17{,}250) - 0}{\sqrt{\frac{(1{,}981)^2}{50} + \frac{(1{,}865)^2}{30}}} = \frac{-654}{440.94} = -1.48$$

$$\sigma_{(\bar{x}_1 - \bar{x}_2)} = \sqrt{\frac{{\sigma_1}^2}{n_1} + \frac{{\sigma_2}^2}{n_2}} \approx \sqrt{\frac{{s_1}^2}{n_1} + \frac{{s_2}^2}{n_2}}$$

Independent samples

The test is valid if:

- The two samples are randomly selected in an independent manner from two target populations.
- The sample sizes, n_1 and n_2, are both large (i.e., $n_1 \geq 30$ and $n_2 \geq 30$).

What if these conditions are violated:

- Test for paired samples
- t-test for small samples

Hypothesis testing - two samples: practice

Many psychologists believe that knowledge of a student’s relationship with his or her parents can be useful in predicting the students’ future interpersonal relationships both on the job and in private life. Researchers at the University of South Alabama compared attitudes of male and female students toward their fathers (*Journal of Genetic Psychology*, March 1998). Using a five-point Likert-type scale, they asked each group to complete the following statement: My relationship with my father can best be described as (1) Awful! (2) Poor, (3) Average, (4) Good, or (5) Great! The data are listed in the file FATHER.

- Using Stata establish if male students tend to have better relationships, on average, with their fathers than female students. Conduct the appropriate hypothesis test using $\alpha = 0.01$. (for testing use variable ‘gender’ with numeric codes for gender categories: female=1, male=2)

Paired samples

Example 1.

You work as a manager of a restaurant. You want to compare performance of your restaurant to that of your competitor. You record the restaurants' total sales for each of 12 randomly selected days during a six-month period. Test whether the data provide evidence of a difference between the mean daily sales of the two restaurants.

Paired samples

OBS	DAY	SALES1	SALES2
1	WED	1005	918
2	SAT	2073	1971
3	TUE	873	825
4	WED	1074	999
5	FRI	1932	1827
6	THR	1338	1281
7	THR	1449	1302
8	MON	759	678
9	FRI	1905	1782
10	MON	693	639
11	SAT	2106	2049
12	TUE	981	933

Paired samples

Note that:

- Samples are not independent
- Samples are small

Therefore we should use ***<u>t-test for paired samples</u>***

Paired samples

	OBS	DAY	SALES1	SALES2	Difference = SALES1 - SALES2
1	OBS	DAY	SALES1	SALES2	Difference = SALES1 - SALES2
2	1	WED	1005	918	87
3	2	SAT	2073	1971	102
4	3	TUE	873	825	48
5	4	WED	1074	999	75
6	5	FRI	1932	1827	105
7	6	THR	1338	1281	57
8	7	THR	1449	1302	147
9	8	MON	759	678	81
10	9	FRI	1905	1782	123
11	10	MON	693	639	54
12	11	SAT	2106	2049	57
13	12	TUE	981	933	48

$$\bar{d} = 82$$

$$s_d = 32$$

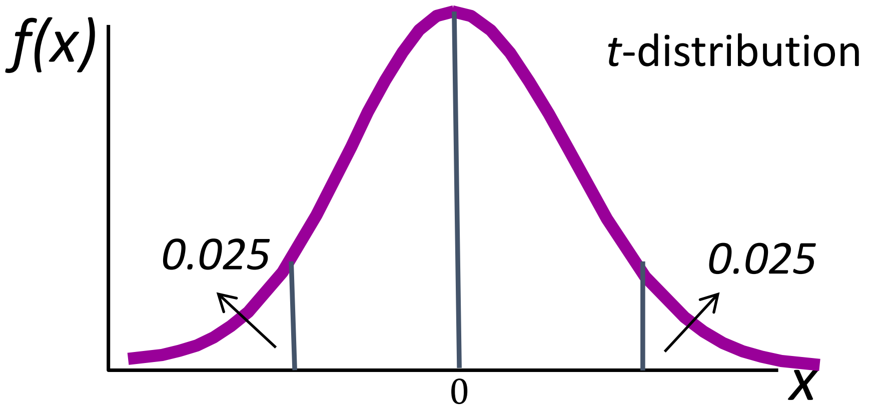
Paired samples

$H_0: \mu_d = 0 \; [i.e. \; (\mu_1 - \mu_2) \; = 0]$

$H_a: \mu_d \neq 0 \; [i.e. \; (\mu_1 - \mu_2) \neq 0]$

Test statistic

$$t = \frac{\overline{d} - 0}{s_{\overline{d}}/\sqrt{n}}$$



$\alpha = 0.05$

where $\overline{d} = sample\ mean\ difference$

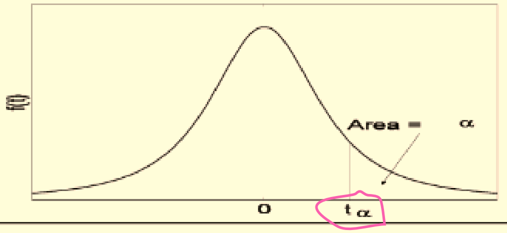
$s_d = sample\ standard\ deviation\ of\ differences$

$n_d = number\ of\ differences = number\ of\ pairs$

$$t = \frac{82 - 0}{32/\sqrt{12}} = \frac{82}{9.238} = 8.88$$

d.f. = (n – 1)

d.f. = 12 – 1 = 11



d.f.	α						
	0.100	0.050	0.025	0.010	0.005	0.001	0.0005
1	3.0777	6.3138	12.7062	31.8205	63.6567	318.3088	636.6193
2	1.8856	2.9200	4.3027	6.9646	9.9248	22.3271	31.5991
3	1.6377	2.3534	3.1824	4.5407	5.8409	10.2145	12.9240
4	1.5332	2.1318	2.7764	3.7469	4.6041	7.1732	8.6103
5	1.4759	2.0150	2.5706	3.3649	4.0321	5.8934	6.8688
6	1.4398	1.9432	2.4469	3.1427	3.7074	5.2076	5.9588
7	1.4149	1.8946	2.3646	2.9980	3.4995	4.7853	5.4079
8	1.3968	1.8595	2.3060	2.8965	3.3554	4.5008	5.0413
9	1.3830	1.8331	2.2622	2.8214	3.2498	4.2968	4.7809
10	1.3722	1.8125	2.2281	2.7638	3.1693	4.1437	4.5869
11	1.3634	1.7959	2.2010	2.7181	3.1058	4.0247	4.4370
12	1.3562	1.7823	2.1788	2.6810	3.0545	3.9296	4.3178
13	1.3502	1.7709	2.1604	2.6503	3.0123	3.8520	4.2208
14	1.3450	1.7613	2.1448	2.6245	2.9768	3.7874	4.1405
15	1.3406	1.7531	2.1314	2.6025	2.9467	3.7328	4.0728
16	1.3368	1.7459	2.1199	2.5835	2.9208	3.6862	4.0150
17	1.3334	1.7396	2.1098	2.5669	2.8982	3.6458	3.9651
18	1.3304	1.7341	2.1009	2.5524	2.8784	3.6105	3.9216
19	1.3277	1.7291	2.0930	2.5395	2.8609	3.5794	3.8834
20	1.3253	1.7247	2.0860	2.5280	2.8453	3.5518	3.8495
21	1.3232	1.7207	2.0796	2.5176	2.8314	3.5272	3.8193
22	1.3212	1.7171	2.0739	2.5083	2.8188	3.5050	3.7921
23	1.3195	1.7139	2.0687	2.4999	2.8073	3.4850	3.7676
24	1.3178	1.7109	2.0639	2.4922	2.7969	3.4668	3.7454
25	1.3163	1.7081	2.0595	2.4851	2.7874	3.4502	3.7251
26	1.3150	1.7056	2.0555	2.4786	2.7787	3.4350	3.7066
27	1.3137	1.7033	2.0518	2.4727	2.7707	3.4210	3.6896
28	1.3125	1.7011	2.0484	2.4671	2.7633	3.4082	3.6739
29	1.3114	1.6991	2.0452	2.4620	2.7564	3.3962	3.6594
30	1.3104	1.6973	2.0423	2.4573	2.7500	3.3852	3.6460
40	1.3031	1.6839	2.0211	2.4233	2.7045	3.3069	3.5510
60	1.2958	1.6706	2.0003	2.3901	2.6603	3.2317	3.4602
120	1.2886	1.6577	1.9799	2.3578	2.6174	3.1595	3.3735
∞	1.2816	1.6449	1.9600	2.3263	2.5758	3.0902	3.2905

Paired samples

$H_0: \mu_d = 0 \ [i.e.\ (\mu_1 - \mu_2) \ = 0]$

$H_a: \mu_d \neq 0 \ [i.e.\ (\mu_1 - \mu_2) \neq 0]$

Test statistic $t = \dfrac{\overline{d} - 0}{s_{\overline{d}}/\sqrt{n}}$

where $\overline{d} = sample\ mean\ difference$

$s_d = sample\ standard\ deviation\ of\ differences$

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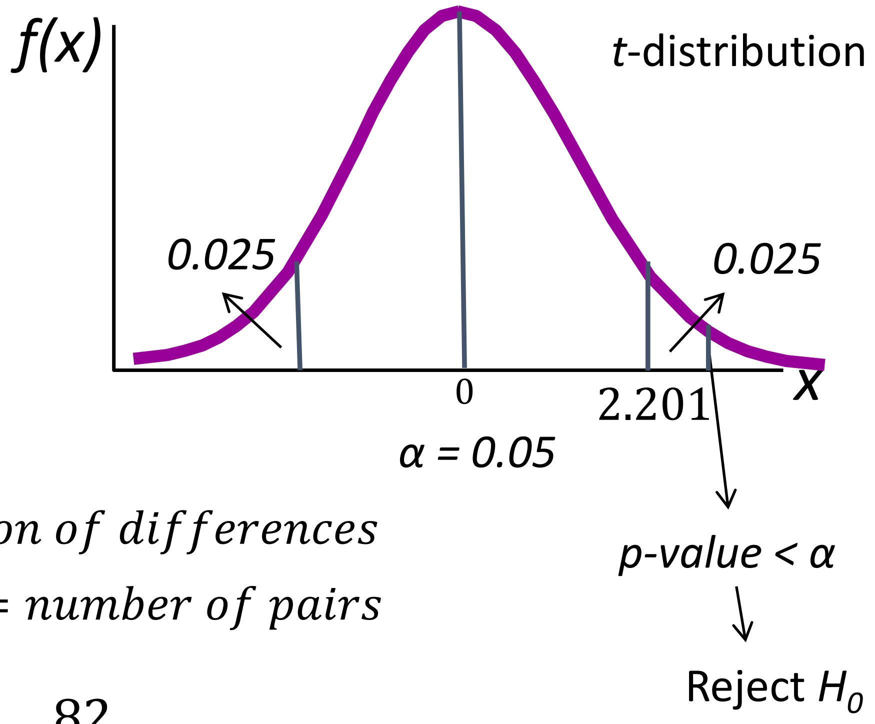

$$t = \frac{82 - 0}{32/\sqrt{12}} = \frac{82}{9.238} = 8.88$$