TRAINING COURSES ON APPLIED ECONOMETRIC ANALYSIS (SUMMER SCHOOL) FOR YOUNG ECONOMISTS / RESEARCHERS ORGANIZED BY WIUT AND IFPRI
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Basics of Probability theory

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Outline

- **Session 1 Basic probability concepts**
  - (Tue 13 June at 13:30-15:00)
- **Session 2 Basic probability concepts (continued)**
  - (Tue 13 June at 15:30-17:00)
- **Session 3 Probability distributions (Discrete)**
  - (Wed 14 June at 9:00-10:30)
- **Session 4 Probability distributions (Continuous)**
  - (Wed 14 June at 11:00-12:30)

Sessions 1-2 Basic probability concepts

- Operations on sets (Union, Intersection and Complement)
- Concepts of experiment, outcome, event,
- Approaches to probability (Classical and Empirical)
- Relationships of events (Mutually Exclusive, Dependent and Independent, Joint probability, Bayes' Formula)

Set

Set is a finite (or countable) or infinite (uncountable) collection of objects (called elements) in which order has no significance.

Examples: $A = \{1,2\} = \{2,1\}$, $P = \{pen, paper, book\}$, $N = \{1,2,3,4, \dots\}$.

If x is an element of A, then $x \in A$. If A is a subset of B, then $A \subset B$.

The size (number of elements) of the set S is indicated by: $n(S)$.

Examples: $2 \in A, 5 \notin A$. $A \subset N$, $N \not\subset A$. $n(A) = 2, n(N) = +\infty$.

Ways of presenting sets:

Enumeration: $A = \{x_1, x_2, \dots, x_n\}$ Description: $B = \{x | property\}$

Examples: Enumeration: $D = \{1,2,3\}$ Description: $D = \{x \in N | 1 \leq x \leq 3\}$

Exercise: Write the roots of $x^2 - 3x + 2 = 0$ in a set notation.

Operations on sets

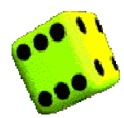
$\boldsymbol{A} \cup \boldsymbol{B}$ $(A\ union\ B)$ holds the elements of A or B or both. $\{x|x \in A\ or\ x \in B\}$

$\boldsymbol{A} \cap \boldsymbol{B}$ $(A\ intersection\ B)$ holds the elements of both A and B. $\{x|x \in A\ and\ x \in B\}$

$\boldsymbol{A}'\ or\ \boldsymbol{A}^{\boldsymbol{C}}$ $(A\ compliment)$ holds the elements that are not in A. $\{x|x \in U, x \notin A\}$

$\boldsymbol{A} \backslash \boldsymbol{B}$ $(difference)$ holds the elements that belong to A but not to B. $\{x|x \in A, x \notin B\}$

Example: Tossing a six-sided die.



Universal set: $U = \{1,2,3,4,5,6\}$.

Subsets: $E = \{2,4,6\}$, $O = \{1,3,5\}$, $P = \{2,3,5\}$.

$E \cup P = \{2,3,4,5,6\}$; $O \cap P = \{3,5\}$; $E' = \{1,3,5\} = O$; $E \backslash P = \{4,6\}$.

Note: $E \cap O = \emptyset$. Note: $E \cup E' = U$.

Exercise: Find: 1) $O \cup P$; 2) $E \cap P$; 3) P'; 4) $O \backslash P$.

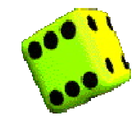
Homework: De Morgan's laws: 1) $(E \cup P)' = E' \cap P'$; 2) $(E \cap O)' = E' \cup O'$.

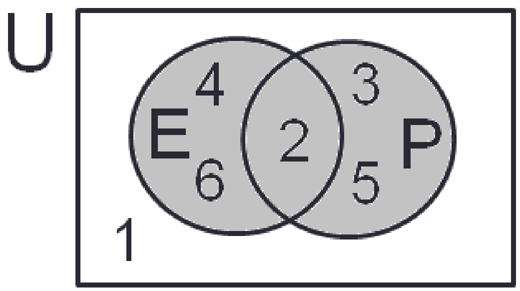
Venn diagram

Venn diagram is a graphical method to illustrate the set operations.

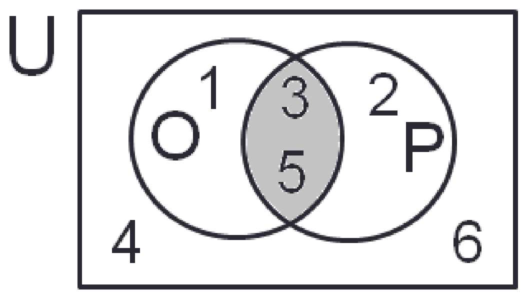
Example: Tossing a six-sided die.

Universal set: $U = \{1,2,3,4,5,6\}$. Subsets: $E = \{2,4,6\}$, $O = \{1,3,5\}$, $P = \{2,3,5\}$.

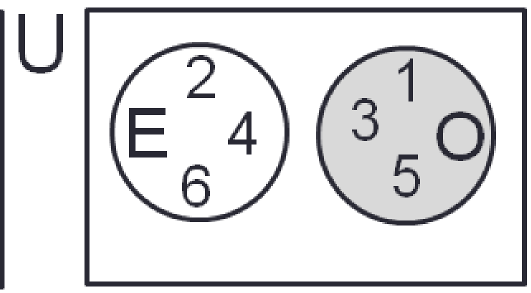




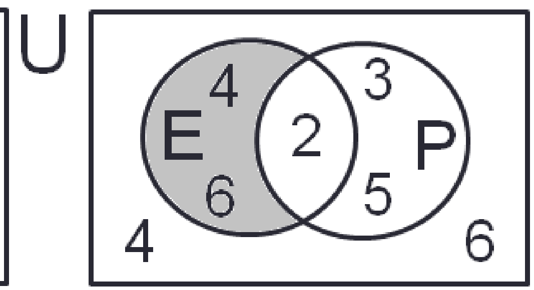
$E \cup P = \{2,3,4,5,6\}$



$O \cap P = \{3,5\}$;
Note: $E \cap O = \emptyset$.



$E' = \{1,3,5\} = O$;
Note: $E \cup E' = U$.



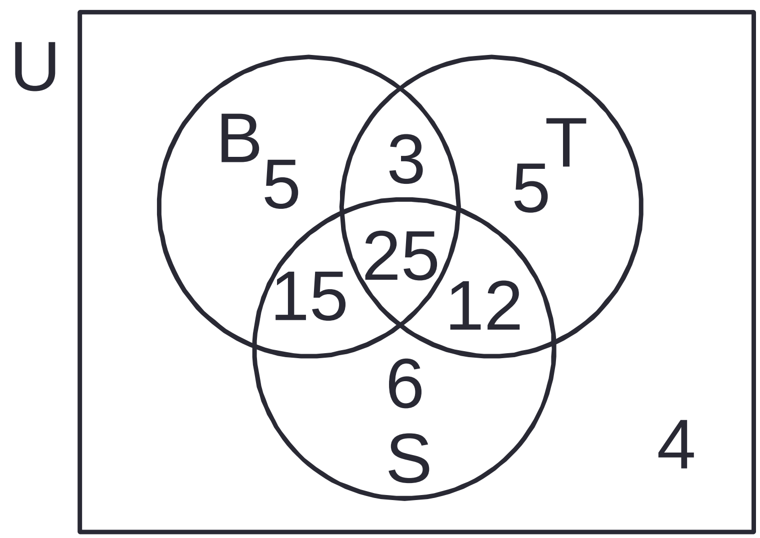
$E \backslash P = \{4,6\}$.

Formula: $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Example: $n(E \cup P) = n(E) + n(P) - n(E \cap P) = 3 + 3 - 1 = 5$.
Exercise: Find: $n(O \cap P)$.

Exercise: Venn diagram

75 students were surveyed, 48 like basketball, 45 like tennis, 58 like swimming, 28 like basketball and tennis, 37 like tennis and swimming, 40 like basketball and swimming and 25 like all 3 sports. How many students do not like any of the sports?



Given:

$B = 48$

$T = 45$

$S = 58$

$B \cap T = 28$

$T \cap S = 37$

$B \cap S = 40$

$B \cap T \cap S = 25$

Thus: $U \backslash (B \cup T \cup S) = 75 - (5 + 3 + 5 + 12 + 25 + 15 + 6) = 4$

Probability

Any activity or process involves a chance (or probability, possibility, likelihood)

Examples:

- Forecasting (chance of raining or not)
- Lottery or any game (chance of winning, losing or making a draw)
- Running a business (chance of succeeding or failing)
- Investing (chance of making extra money or not)
- Buying a product (chance of buying normal or defective product)
- Taking an exam (chance of passing or failing)

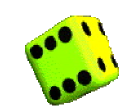
Exercise: You give an example

Experiment and Sample space

Experiment is any activity or process that generates well-defined outcomes (or results).

Sample space for an experiment is the set of all experimental outcomes.

Experiment	Experimental outcomes	Sample space
Toss a coin	Head, Tail	S = {Head, Tail}
Apply for a job	Hired, Not hired	S = {Hired, Not Hired}
Roll a die	1, 2, 3, 4, 5, 6	S = {1, 2, 3, 4, 5, 6}
Run a business	Profit, Loss, Even	S = {Profit, Loss, Even}
Sales (units)	0, 1, 2, 3, 4, …	S = {0,1,2,3,4,…}



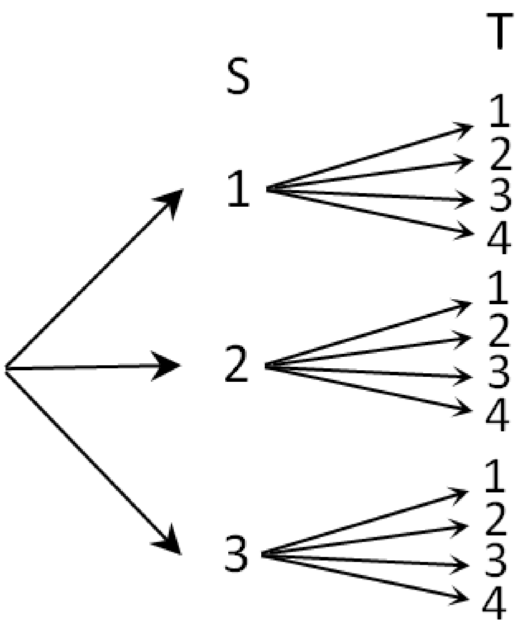
Counting principles of experiments

- **Multiple-step**
- **Combination (abc)**
- **Permutation (abc, acb, bac, bca, cab, cba)**

Multiple-step experiment

- k **steps with a certain number of outcomes at each step.**
- **Total number of outcomes** $= n_1 \cdot n_2 \cdots n_k$.

Example: A man has 3 kinds of shirts and 4 kinds of ties. How many different options does he have to wear them?



Total number of outcomes $= n_1 \cdot n_2 = 3 \cdot 4 = 12$

$$S = \begin{Bmatrix} S1T1, S1T2, S1T3, S1T4, \\ S2T1, S2T2, S2T3, S2T4, \\ S3T1, S3T2, S3T3, S3T4 \end{Bmatrix}$$

Exercise:

1) How many total outcomes are there if his wife has 6 shirts and 5 ties?

2) How many total outcomes are there for the husband and wife altogether?

Combination

- **n items must be selected from a set of N items, where the order of selection does not matter.**
- **Combinations of N items taken n at a time:**

$$C_n^N = \binom{N}{n} = \frac{N!}{n!(N-n)!}, \text{ where } n! = 1 \cdot 2 \cdot 3 \cdots n$$

Example: How many ways can 2 out of 5 employees (A, B, C, D, E) be selected for a project?

Solution: $N = 5$ and $n = 2$. Total number of outcomes:

$$C_2^5 = \frac{5!}{2!\,(5-2)!} = \frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5}{1 \cdot 2 \cdot 1 \cdot 2 \cdot 3} = 10$$

S = {AB, AC, AD, AE,   BC, BD, BE,   CD, CE,   DE}

Permutation

- **n items must be selected from a set of N items, where the order of selection is important**
- **Permutations of N items taken n at a time:**

$$P_n^N = \frac{N!}{(N-n)!}, \text{ (compare } C_n^N = \frac{N!}{n!(N-n)!}\text{)}$$

Example: How many ways can a director and a secretary be chosen out of 5 employees (A, B, C, D, E)?

Solution: N = 5 and n = 2. Total number of outcomes:

$$P_2^5 = \frac{5!}{(5-2)!} = \frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5}{1 \cdot 2 \cdot 3} = 20$$

S = {AB, AC, AD, AE; BA, BC, BD, BE; CA, CB, CD, CE;
DA, DB, DC, DE; EA, EB, EC, ED}

Exercises

1) An investor has two stocks: stock A and stock B. Each stock may increase in value, decrease in value, or remain unchanged. Consider the experiment of investing in the two stocks and observing the change (if any) in value. How many experimental outcomes are possible?
2) Suppose there are 8 runners in a race. How many ways can awards for the 1st, 2nd, and 3rd places be presented?
3) A computer programming team has 12 members. How many ways can a group of 7 (out of the 12 members) be selected to work on a special project?

Answers: ______________________

Event

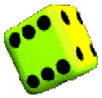
Event is a subset of the sample space. Alternatively, it is a collection (group) of outcomes.

Simple event contains a single outcome.

Example:

The experiment of rolling a die has a sample space of six outcomes:

$$S = \{1, 2, 3, 4, 5, 6\}.$$



1) Simple events: $E_1 = \{1\}$, $E_2 = \{2\}$, $E_3 = \{3\}$, $E_4 = \{4\}$, $E_5 = \{5\}$, $E_6 = \{6\}$.
2) Event of an even number: $A = \{2, 4, 6\}$.
3) Event of an odd number: $B = \{1, 3, 5\}$.