Identification: Difference-in-Difference estimator

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Today's Class

- Non-experimental Methods: Difference-in-differences
  - Understanding how it works
  - How to test the assumptions
  - Some problems and pitfalls

Why are experiments good?

- Treatment is *random* so it's independent of other characteristics

- This independence allows us to develop an *implied counterfactual*

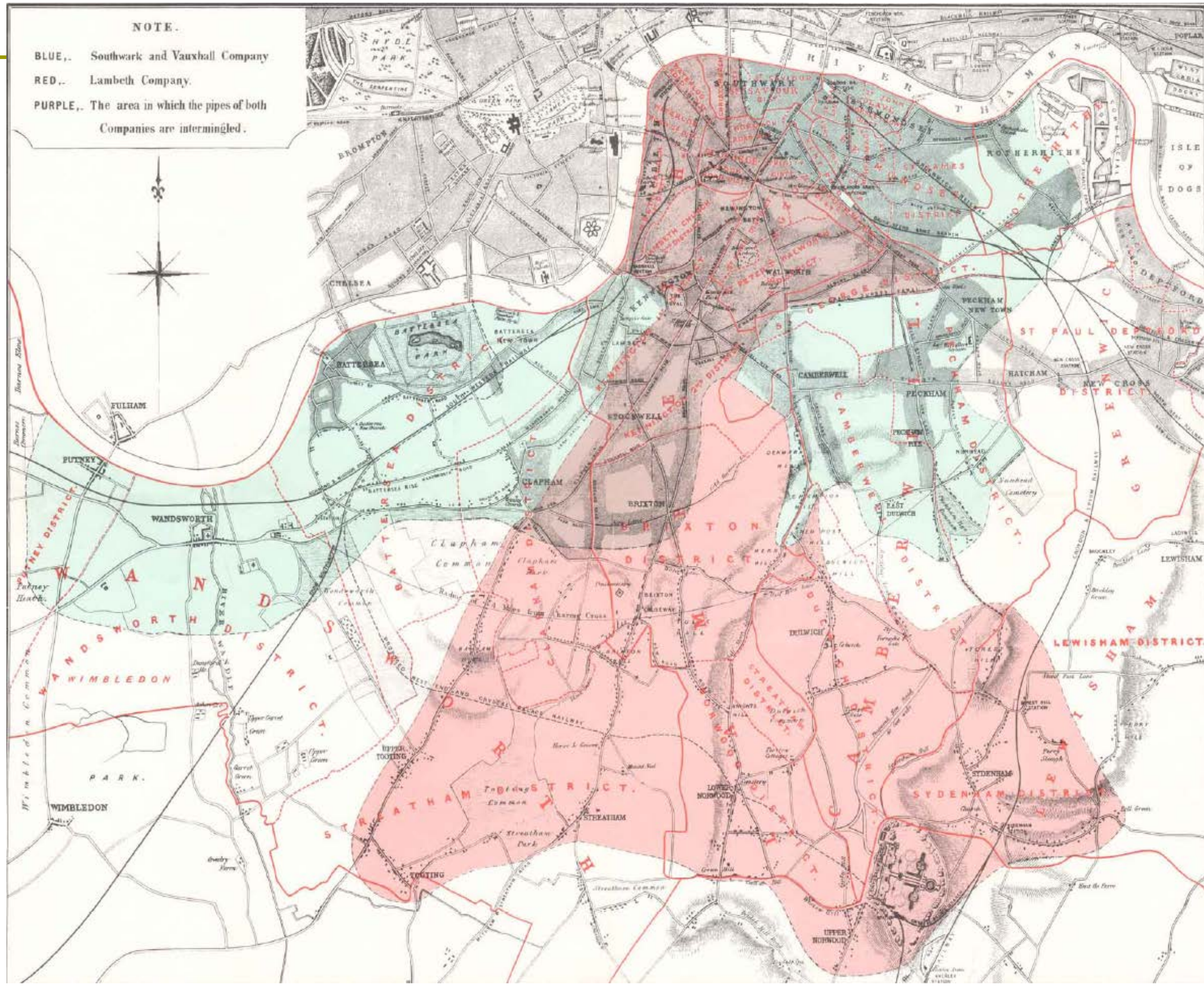
- Thus even though we don't observe $E[Y_0 | T=1]$ we can use $E[Y_0 | T=0]$ as the counterfactual for the treatment group

What if we don’t have an experiment

- Would like to find a group that is exactly like the treatment group but didn’t get the treatment

- Hard to do because
  - Lots of unobservables
  - Data is limited
  - Selection into treatment

John Snow


NOTE.
BLUE,. Southwark and Vauxhall Company
RED,. Lambeth Company.
PURPLE,. The area in which the pipes of both Companies are intermingled.
SOUTHWARK
BERMONDSEY
ROTHERHITHE
ISLE OF DOGS
KENNINGTON
WALWORTH
CHELSEA
BATTERSEA
FULHAM
PUTNEY
WANDSWORTH
CLAPHAM
STOCKWELL
BRIXTON
CAMBERWELL
PECKHAM
DULWICH
STREATHAM
TOOTING
WIMBLEDON
LOWER NORWOOD
UPPER NORWOOD
SYDENHAM
LEWISHAM
LEWISHAM DISTRICT.
NEW CROSS
HATCHAM

Background Information

- Water supplied to households by competing private companies
- Sometimes different companies supplied households in same street
- In south London two main companies:
  - Lambeth Company (water supply from Thames Ditton, 22 miles upstream)
  - Southwark and Vauxhall Company (water supply from Thames)

In 1853/54 cholera outbreak

- Death Rates per 10000 people by water company
  - Lambeth 10
  - Southwark and Vauxhall 150

- Might be water but perhaps other factors

- Snow compared death rates in 1849 epidemic
  - Lambeth 150
  - Southwark and Vauxhall 125

- In 1852 Lambeth Company had changed supply from Hungerford Bridge

The effect of clean water on cholera death rates

	1849	1853/54	Difference
Lambeth	150	10	-140
Vauxhall and Southwark	125	150	25
Difference	-25	140	-165

Counterfactual 2: 'Control' group time difference. Assume this would have been true for 'treatment' group

Counterfactual 1: Pre-Experiment difference between treatment and control—assume this difference is *fixed* over time

This is basic idea of Differences-in-Differences

- Have already seen idea of using differences to estimate causal effects
  - Treatment/control groups in experimental data

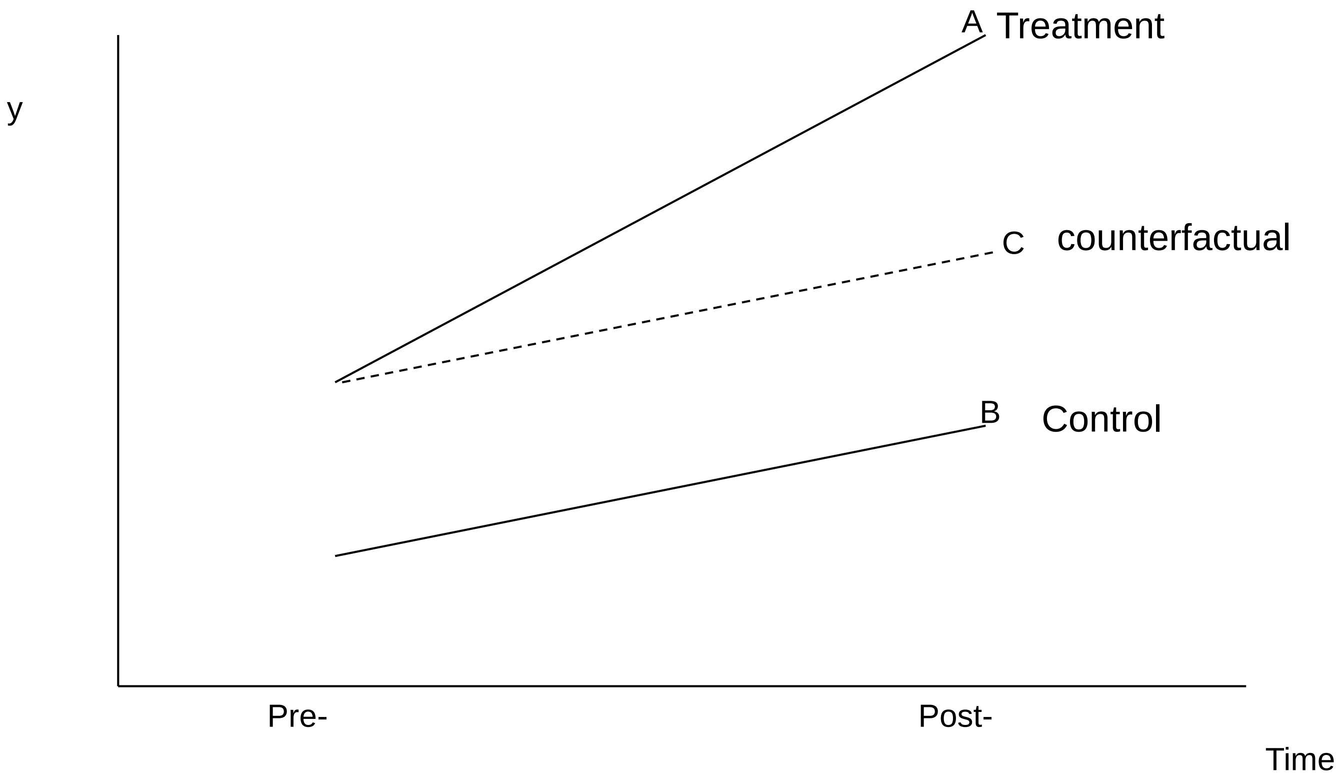
- We need a counterfactual because we don't observe the outcome of the treatment group when they weren't treated (i.e. $(Y_0 \mid T=1)$)

- Often would like to find 'treatment' and 'control' group who can be assumed to be similar in every way except receipt of treatment

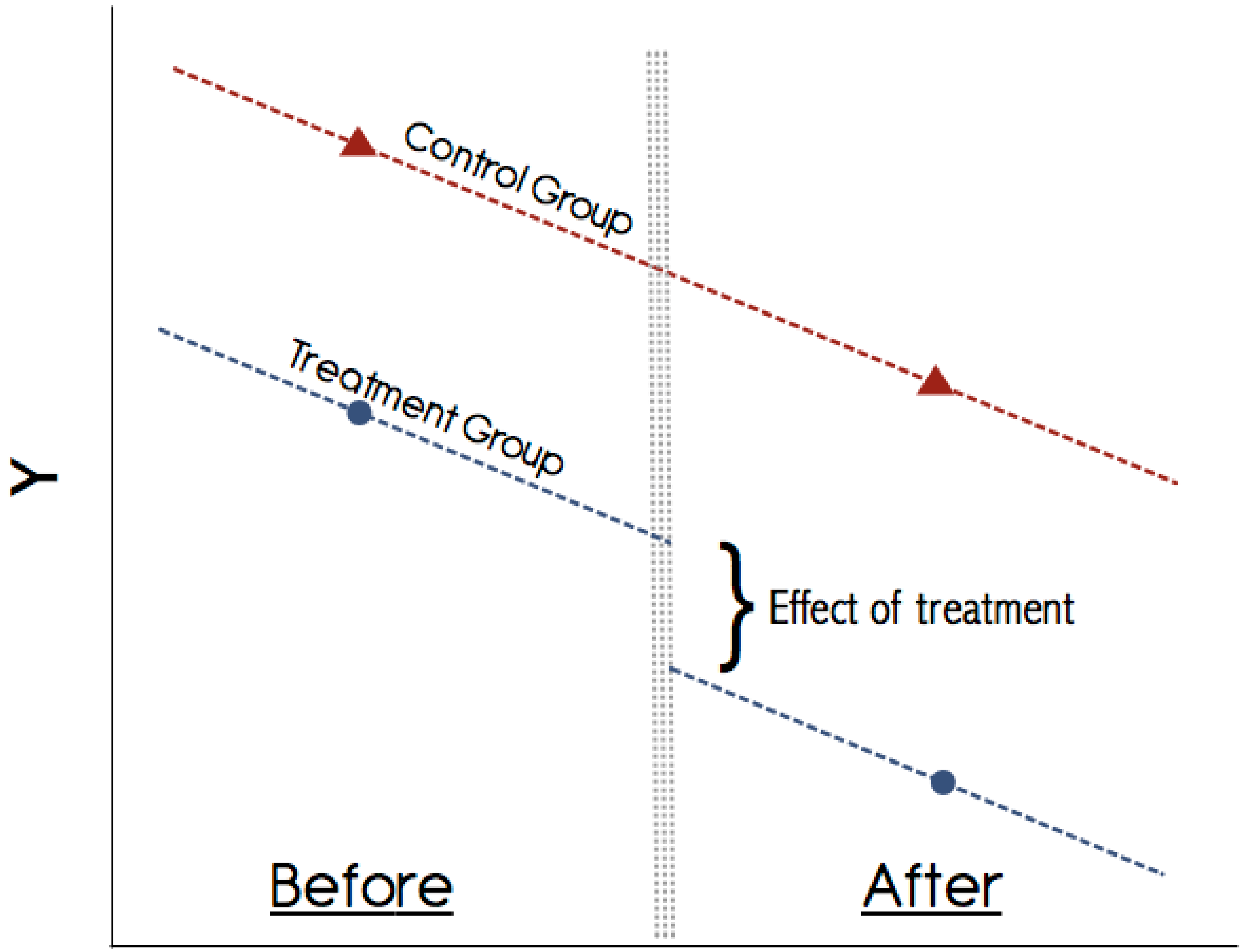
A Weaker Assumption is..

- Assume that, in absence of treatment, difference between 'treatment' and 'control' group is constant over time
- With this assumption can use observations on treatment and control group pre- and post-treatment to estimate causal effect
- Idea
  - Difference pre-treatment is 'normal' difference
  - Difference pre-treatment is 'normal' difference + causal effect
  - Difference-in-difference is causal effect

A Graphical Representation



A – B = Standard differences estimator
C – B = Counterfactual ‘normal’ difference
A – C = Difference-in-Difference Estimate


Control Group
Treatment Group
Effect of treatment
Y
Before
After

Assumption of the D-in-D estimate

- D-in-D estimate assumes trends in outcome variables the same for treatment and control groups
  - Fixed difference over time
  - This is not testable because we never observe the counterfactual

- Is this reasonable?
  - With two periods can’t do anything
  - With more periods can see if control and treatment groups ‘trend together’

Some Notation

- Define:

$$\mu_{it} = \mathrm{E}(y_{it})$$

Where i=0 is control group, i=1 is treatment

Where t=0 is pre-period, t=1 is post-period

- Standard 'differences' estimate of causal effect is estimate of:

$$\mu_{11} - \mu_{01}$$

- 'Differences-in-Differences' estimate of causal effect is estimate of:

$$(\mu_{11} - \mu_{01}) - (\mu_{10} - \mu_{00})$$

How to estimate?

- Can write D-in-D estimate as:

$$\underbrace{(\mu_{11} - \mu_{10})}_{\text{Before-After difference for 'treatment' group}} - \underbrace{(\mu_{01} - \mu_{00})}_{\text{Before-After difference for 'control' group}}$$

- This is simply the difference in the change of treatment and control groups so can estimate as:

$$\Delta y_i = \beta(\Delta X_i) + \Delta\varepsilon_i$$

Can we do this?

- This is simply ‘differences’ estimator applied to the difference
- To implement this need to have repeat observations on the same individuals
- May not have this – individuals observed pre- and post-treatment may be different

In this case can estimate….

$$y_{it} = \beta_0 + \beta_1 X_i + \beta_2 T_t + \beta_3 X_i * T_t + \varepsilon_{it}$$

Main effect of Treatment group

(in before period because T=0)

Main effect of the After period

(for control group because X=0)

D-in-D estimate

- D-in-D estimate is estimate of β_3
- why is this?

$$p\lim \hat{\beta}_0 = \mu_{00}$$

$$p\lim \hat{\beta}_1 = \mu_{10} - \mu_{00}$$

$$p\lim \hat{\beta}_2 = \mu_{01} - \mu_{00}$$

$$p\lim \hat{\beta}_3 = \left(\mu_{11} - \mu_{01}\right) - \left(\mu_{10} - \mu_{00}\right)$$

A Comparison of the Two Methods

- Where have repeated observations could use both methods

- Will give same parameter estimates

- But will give different standard errors
  - ‘levels’ version will assume residuals are independent – unlikely to be a good assumption
  - Can deal with this by clustering by group (imposes a covariance structure within the clustering variable)

Recap: Assumptions for Diff-in-Diff

- Additive structure of effects.
  - We are imposing a linear model where the group or time specific effects only enter additively.
- No spillover effects
  - The treatment group received the treatment and the control group did not
- Parallel time trends:
  - there are fixed differences over time.
  - If there are differences that vary over time then our second difference will still include a time effect.

Issue 1: Other Regressors

- Can put in other regressors just as usual
  - think about way in which they enter the estimating equation
  - E.g. if level of W affects level of y then should include ΔW in differences version
- Conditional comparisons might be useful if you think some groups may be more comparable or have different trends than others

Issue 2: Differential Trends in Treatment and Control Groups

- Key assumption underlying validity of D-in-D estimate is that differences between treatment and control group would have remained constant in absence of treatment
  - Can never test this
  - With only two periods can get no idea of plausibility
  - But can with more than two periods

Differences-in-Differences: Summary

- A very useful and widespread approach
- Validity does depend on assumption that trends would have been the same in absence of treatment
- Often need more than 2 periods to test:
  - Pre-treatment trends for treatment and control to see if “fixed differences” assumption is plausible or not
  - See if there’s an Ashenfelter Dip