Difference-in-Difference estimator

Presented at Summer School 2015
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June 9, 2015
Tashkent
Today’s Class

• Non-experimental Methods: Difference-in-differences
  • Understanding how it works
  • How to test the assumptions
  • Some problems and pitfalls
Why are experiments good?

• Treatment is *random* so it’s independent of other characteristics

• This independence allows us to develop an *implied counterfactual*

• Thus even though we don’t observe $\mathbb{E}[Y_0 | T=1]$ we can use $\mathbb{E}[Y_0 | T=0]$ as the counterfactual for the treatment group
What if we don’t have an experiment

• Would like to find a group that is exactly like the treatment group but didn’t get the treatment
• Hard to do because
  • Lots of unobservables
  • Data is limited
  • Selection into treatment
Background Information

• Water supplied to households by competing private companies

• Sometimes different companies supplied households in same street

• In south London two main companies:
  • Lambeth Company (water supply from Thames Ditton, 22 miles upstream)
  • Southwark and Vauxhall Company (water supply from Thames)
In 1853/54 cholera outbreak

• Death Rates per 10000 people by water company
  • Lambeth 10
  • Southwark and Vauxhall 150
• Might be water but perhaps other factors
• Snow compared death rates in 1849 epidemic
  • Lambeth 150
  • Southwark and Vauxhall 125
• In 1852 Lambeth Company had changed supply from Hungerford Bridge
The effect of clean water on cholera death rates

<table>
<thead>
<tr>
<th></th>
<th>1849</th>
<th>1853/5 4</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lambeth</td>
<td>150</td>
<td>10</td>
<td>-140</td>
</tr>
<tr>
<td>Vauxhall and Southwark</td>
<td>125</td>
<td>150</td>
<td>25</td>
</tr>
<tr>
<td>Difference</td>
<td>-25</td>
<td>140</td>
<td>-165</td>
</tr>
</tbody>
</table>

Counterfactual 1: Pre-Experiment difference between treatment and control—assume this difference is fixed over time

Counterfactual 2: ‘Control’ group time difference. Assume this would have been true for ‘treatment’ group
This is basic idea of D-i-D

- Have already seen idea of using differences to estimate causal effects
  - Treatment/control groups in experimental data
- We need a counterfactual because we don’t observe the outcome of the treatment group when they weren’t treated (i.e. \((Y_0 / T=1)\))
- Often would like to find ‘treatment’ and ‘control’ group who can be assumed to be similar in every way except receipt of treatment
A Weaker Assumption is..

- Assume that, in absence of treatment, difference between ‘treatment’ and ‘control’ group is constant over time.
- With this assumption can use observations on treatment and control group pre- and post-treatment to estimate causal effect.

- Idea
  - Difference pre-treatment is ‘normal’ difference
  - Difference pre-treatment is ‘normal’ difference + causal effect
  - Difference-in-difference is causal effect
A Graphical Representation

$A - B = \text{Standard differences estimator}$

$C - B = \text{Counterfactual ‘normal’ difference}$

$A - C = \text{Difference-in-Difference Estimate}$
Assumption of the D-in-D estimate

- D-in-D estimate assumes trends in outcome variables the same for treatment and control groups
  - Fixed difference over time
  - This is not testable because we never observe the counterfactual

- Is this reasonable?
  - With two periods can’t do anything
  - With more periods can see if control and treatment groups ‘trend together’
Some Notation

• Define:

\[ \mu_{it} = E(y_{it}) \]

Where i=0 is control group, i=1 is treatment
Where t=0 is pre-period, t=1 is post-period

• Standard ‘differences’ estimate of causal effect is estimate of:

\[ \mu_{11} - \mu_{01} \]

• ‘Differences-in-Differences’ estimate of causal effect is estimate of:

\[ (\mu_{11} - \mu_{01}) - (\mu_{10} - \mu_{00}) \]
How to estimate?

• Can write D-in-D estimate as:

\[ (\mu_{11} - \mu_{10}) - (\mu_{01} - \mu_{00}) \]

Before-After difference for ‘treatment’ group

Before-After difference for ‘control’ group

• This is simply the difference in the change of treatment and control groups so can estimate as:

\[ \Delta y_i = \beta(\Delta X_i) + \Delta \varepsilon_i \]
Can we do this?

• This is simply ‘differences’ estimator applied to the difference

• To implement this need to have repeat observations on the same individuals

• May not have this – individuals observed pre- and post-treatment may be different
In this case can estimate....

\[ y_{it} = \beta_0 + \beta_1 X_i + \beta_2 T_t + \beta_3 X_i \times T_t + \epsilon_{it} \]

Main effect of Treatment group
(in before period because \( T=0 \))

Main effect of the After period
(for control group because \( X=0 \))
D-in-D estimate

• D-in-D estimate is estimate of $\beta_3$
• why is this?

\[
\begin{align*}
\hat{p} \lim \hat{\beta}_0 &= \mu_{00} \\
\hat{p} \lim \hat{\beta}_1 &= \mu_{10} - \mu_{00} \\
\hat{p} \lim \hat{\beta}_2 &= \mu_{01} - \mu_{00} \\
\hat{p} \lim \hat{\beta}_3 &= (\mu_{11} - \mu_{01}) - (\mu_{10} - \mu_{00})
\end{align*}
\]
A Comparison of the Two Methods

• Where have repeated observations could use both methods
• Will give same parameter estimates
• But will give different standard errors
  • ‘levels’ version will assume residuals are independent – unlikely to be a good assumption
  • Can deal with this by clustering by group (imposes a covariance structure within the clustering variable)
Recap: Assumptions for Diff-in-Diff

• Additive structure of effects.
  • We are imposing a linear model where the group or time specific effects only enter additively.

• No spillover effects
  • The treatment group received the treatment and the control group did not

• Parallel time trends:
  • there are fixed differences over time.
  • If there are differences that vary over time then our second difference will still include a time effect.
Issue 1: Other Regressors

• Can put in other regressors just as usual
  • think about way in which they enter the estimating equation
  • E.g. if level of W affects level of y then should include $\Delta W$ in differences version

• Conditional comparisons might be useful if you think some groups may be more comparable or have different trends than others
Issue 2: Differential Trends in Treatment and Control Groups

• Key assumption underlying validity of D-in-D estimate is that differences between treatment and control group would have remained constant in absence of treatment
  • Can never test this
  • With only two periods can get no idea of plausibility
  • But can with more than two periods
An Example:


• Interested in effect of vertical integration on retail petrol prices

• Investigates take-over in CA of independent ‘Thrifty’ chain of petrol stations by ARCO (more integrated)
  • Treatment Group: petrol stations < 1mi from ‘Thrifty’
  • Control group: petrol stations > 1mi from ‘Thrifty’

• Lots of reasons why these groups might be different so D-in-D approach seems a good idea
This picture contains relevant information...

- Can see D-in-D estimate of +5c per gallon

- Also can see trends before and after change very similar – D-in-D assumption valid
Issue 3: Ashenfelter’s Dip

• `pre-program dip', for participants
  • Related to the idea of *mean reversion*: individuals experience some idiosyncratic shock
  • May enter program when things are especially bad
  • Would have improved anyway (reversion to the mean)

• Another issue may be if your treatment is *selected* by participants then only the worst off individuals elect the treatment—not comparable to general effect of policy
Another Example...

• Interested in effect of government-sponsored training (MDTA) on earnings

• Treatment group are those who received training in 1964

• Control group are random sample of population as a whole
Earnings for period 1959-69
Things to Note..

- Earnings for trainees very low in 1964 as training not working in that year – should ignore this year

- Simple D-in-D approach would compare earnings in 1965 with 1963

- But earnings of trainees in 1963 seem to show a ‘dip’ – so D-in-D assumption probably not valid

- Probably because those who enter training are those who had a bad shock (e.g. job loss)
Differences-in-Differences: Summary

• A very useful and widespread approach

• Validity does depend on assumption that trends would have been the same in absence of treatment

• Often need more than 2 periods to test:
  • Pre-treatment trends for treatment and control to see if “fixed differences” assumption is plausible or not
  • See if there’s an Ashenfelter Dip
Issues

• Economic effects of minimum wages and evidence on minimum wages and employment

• The controversy on ‘conventional wisdom’ versus micro based ‘revisionist’ approach
Economic Effects of Minimum Wages

- Effect on employment/unemployment has been central issue in debate about economic effects of minimum wages.
- Standard textbook model of labour demand produces one of the clearest predictions in labour economics - minimum wages price workers out of jobs by forcing employers up their labour demand curve.
Standard Textbook Model
Standard Textbook Model

• Basic model rests upon several assumptions: complete coverage; homogeneous labour; competitive labour market; short run and long run impact the same.

• Clear prediction: the minimum wage increase results in reduced employment - the proportional reduction in employment \((\ln E_m - \ln E_0)\) equals the proportionate wage increase \((\ln W_m - \ln W_0)\) times the elasticity of demand \(\eta\).

• Can develop more sophisticated models, but with assumption of perfect competition produce same qualitative predictions.
Two Sector Model

• Basic model can be generalised in various directions. One example is to move to a two sector model - covered/non-covered, set $E0 = 1$, $W0 = 1$.

• Demand for workers in the covered sector depends on the minimum wage, whereas demand in the uncovered sector depends upon the market wage.

• Minimum wage elasticity of employment =

  \[ c\eta\varepsilon\ln Wm / [1 - c + \varepsilon\ln Wm] \]

  where $c = \text{proportion in covered sector}$, $\varepsilon = \text{elasticity of labour supply}$.

• If $c = 1$, $\varepsilon = \infty \Rightarrow \text{standard one sector competitive model}$, $\eta$

• Example: $c = 0.7$, $\ln Wm = 0.6$, $\varepsilon = 0.3$, $\eta = -1 \Rightarrow \text{employment effect} = -0.26$. 
Implications

• Only pertinent question is ‘how negative is the negative effect on employment?’

• Minimum wage hurts the people it sets out to help by pricing them out of work – even more the case since low skill people more likely to be low paid
Evidence

- Early empirical work largely supportive of basic model → ‘conventional wisdom’.
- Usually based on aggregate time series studies of US employment/unemployment rates and minimum wages, usually focussing on teenagers

\[ Y_t = g(MW_t, X_{1t}, \ldots, X_{kt}) + e_t \]

where \( Y_t \) = employment / unemployment to population ratios (usually in logs), \( X_{it} \) = aggregate demand and supply variables (teenagers in training programmes, school enrollment, time trend), \( MW_t \) = minimum wage index (e.g. Kaitz index).

- Brown, Gilroy, Kohen (1982) *Journal of Economic Literature* - say “consensus” reached: minimum wages reduce teenage employment with elasticities in the -0.1 to -0.3 range.
Observations on Time Series Evidence: Re-Appraisal

1). US minimum wage fell strongly in real terms in the 1980s
Extending the samples of teenage employment studies into the 1980s produces much smaller, often statistically insignificant, elasticities below the ‘consensus’ range (around -0.07) (Card and Krueger, 1995).

Minimum wage effects on employment seem small (centring in on zero).

Or – labour demand curve inelastic so that employment not very sensitive to changes in minimum wages.

• What is best conceptual way to evaluate economic effect of minimum wage?

• ‘Before and after’ micro work more closely approximates the theoretical approaches that talk about labour markets with and without minimum wage floors – sometimes referred to as ‘revisionist’ approach.
Methodological Issues in Newer Research

• Corresponds better to theoretical concepts as adopts before and after approach, with treatment and control groups.

• If $E$ is employment, $T$ and $C$ denote treatment and controls and 1 and 2 are the before and after treatment periods then an estimate of the impact of treatment is:

\[
(ET_2 – EC_2) – (ET_1 – EC_1)
\]

or

\[
(ET_2 – ET_1) – (EC_2 – EC_1)
\]

Most famous piece is Card and Krueger’s (1994) New Jersey / Pennsylvania comparison
New Jersey/Pennsylvania Comparison
(Card and Krueger, 1994)

• Can be viewed as case study of fast food industry.
• In April 1992 the New Jersey minimum wage went up from the federal minimum level of $4.25 to $5.05 but the minimum in Pennsylvania remained at $4.25.
Minimum Wages and Employment:
A Case Study of the Fast-Food Industry in
New Jersey and Pennsylvania

David Card and Alan B. Krueger

*American Economic Review* 84(4),
• **Question of Interest**
  “How do employers in a low-wage labor market respond to an increase in the minimum wage?”

• **Approach**
  Compare employment of teenagers in New Jersey and eastern Pennsylvania before and after the increase in the minimum wage in NJ from $4.25 to $5.05 on April 1, 1992.

• **Methods of Analysis**
  • Difference-in-Differences
  • Regression Analysis
Data

- Phone survey of fast-food restaurants in NJ and eastern Penn (personal interview for 28 stores in wave 2)
  - Wave 1: late Feb. and early Mar. 1992
  - Wave 2: Nov. and Dec. 1992
  - Restaurants: Burger King, KFC, Wendy’s and Roy Rogers
  - 371 restaurants were interviewed in both waves

What if employment increased when the MW in NJ increased? Did the increase in MW cause the increase in employment?
• “Correlation does not mean causation.” Could be that econ conds improved, which caused E to go up.

• **Difference-in-Differences Approach**
  
  • Premise – Economic conditions in NJ and eastern Penn are the same.
  
  • Compare E change in NJ before and after the MW change, with the E change in eastern Penn in the same time period.
  
  • If NJ and eastern Penn E grow at the same rate, MW would have had no effect.
  
  • If E in Penn grew or stayed the same, and E in NJ fell → evidence that MW decreases E.
Results of Difference-in-Difference Analysis

Employment in Typical Fast-Food Restaurants (in FTEs)

<table>
<thead>
<tr>
<th></th>
<th>NJ</th>
<th>E Penn</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before change</td>
<td>20.4</td>
<td>23.3</td>
</tr>
<tr>
<td>After change</td>
<td>21.0</td>
<td>21.2</td>
</tr>
<tr>
<td>Difference</td>
<td>0.6</td>
<td>-2.1</td>
</tr>
</tbody>
</table>

**Difference-in-Differences** = 0.6 – (-2.1) = **2.7**

NJ: Employment increased after MW.
E Penn: Employment decreased after MW → depressed economy in E Penn and NJ.

Despite declining economic conditions, in E Penn (and presumably NJ), employment **increased** in NJ.
Regression Analysis

Equation (1a): $E = a + bX + cNJ$

- $E =$ change in employment from wave 1 to wave 2 at a given restaurant
- $X =$ set of characteristics of the restaurant
- $NJ = 1$ if restaurant is in New Jersey;
  $NJ = 0$ if restaurant is in eastern Pennsylvania.

- If $c < 0$, $E$ lower for NJ than for Penn restaurants after the MW
- If $c > 0$, $E$ higher for NJ
- If $c = 0$, no difference in the change in $E$
• Equation (1b): $E = a' + b'X + c'\text{GAP}$

• GAP = 0 for stores in Penn
• GAP = 0 for stores in NJ with $W_1 \gtrapprox 5.15$
• GAP = $(5.05 - W_1) / W_1$ for other stores in NJ
• GAP is the proportional increase in wages needed to meet the new MW.
  • If $c' < 0$, an increase in the required wage hike, results in a negative change in employment.
  • If $c' > 0$, $E$ higher as stores have to pay more to meet the MW requirement.
  • If $c' = 0$, no difference in the change in $E$ for stores that have to increase wages more.
Regression Results

Equation (1a): $E = a + bX + cNJ$
- $c > 0$

Equation (1b): $E = a' + b'X + c'GAP$
- $c' > 0$

Indicates MW is positively associated with $E$.
Results robust across many specification tests.

Conclusion

April, 1992 rise in the MW did not decrease employment of teens in fast-food restaurants in New Jersey.
Why Counter-Intuitive Result?

- Survey data may not accurately measure actual employment. (Neumark and Wachter (2000) use establishment data with same methodology and find a zero or slightly negative effect.)

- Fast-food restaurants may not be representative of low-wage employers, e.g., there may be fewer opportunities to substitute capital for labor.

- Employers may need more time to adjust to minimum wage changes than comparing just before and just after a change.
Why Counter-Intuitive Result?

- Economic health of the fast-food market in E Penn may be an imperfect match for NJ.

- Adjustment may begin before MW takes effect.
Identification of Employment Effects

\[ \Delta E_i = a + bX_i + cNJ_i + e_i \]

\[ \Delta E_i = a' + b'X_i + c'GAP_i + e'_i \]

Where GAP = 0 for P stores and NJ stores with \( W_{1i} \geq $5.05 \) and = \( (5.05 - W_{1i}) / W_{1i} \) for other NJ stores/
### Table 3 — Average Employment Per Store Before and After the Rise in New Jersey Minimum Wage

<table>
<thead>
<tr>
<th>Variable</th>
<th>Stores by state</th>
<th>Stores in New Jersey</th>
<th>Differences within NJ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. FTE employment before, all available observations</td>
<td>23.33 (1.35)</td>
<td>20.44 (0.51)</td>
<td>−2.89 (1.44)</td>
</tr>
<tr>
<td>2. FTE employment after, all available observations</td>
<td>21.17 (0.94)</td>
<td>21.03 (0.52)</td>
<td>−0.14 (1.07)</td>
</tr>
<tr>
<td>3. Change in mean FTE employment</td>
<td>−2.16 (1.25)</td>
<td>0.59 (0.54)</td>
<td>2.76 (1.36)</td>
</tr>
<tr>
<td>4. Change in mean FTE employment, balanced sample of stores</td>
<td>−2.28 (1.25)</td>
<td>0.47 (0.48)</td>
<td>2.75 (1.34)</td>
</tr>
<tr>
<td>5. Change in mean FTE employment, setting FTE at temporarily closed stores to 0</td>
<td>−2.28 (1.25)</td>
<td>0.23 (0.49)</td>
<td>2.51 (1.35)</td>
</tr>
</tbody>
</table>

**Notes:** Standard errors are shown in parentheses. The sample consists of all stores with available data on employment. FTE (full-time-equivalent) employment counts each part-time worker as half a full-time worker. Employment at six closed stores is set to zero. Employment at four temporarily closed stores is treated as missing.

a Stores in New Jersey were classified by whether starting wage in wave 1 equals $4.25 per hour (N = 101), is between $4.26 and $4.99 per hour (N = 140), or is $5.00 per hour or higher (N = 73).

b Difference in employment between low-wage ($4.25 per hour) and high-wage (≥ $5.00 per hour) stores; and difference in employment between midrange ($4.26−$4.99 per hour) and high-wage stores.

c Subset of stores with available employment data in wave 1 and wave 2.

d In this row only, wave-2 employment at four temporarily closed stores is set to 0. Employment changes are based on the subset of stores with available employment data in wave 1 and wave 2.
# Employment Models

## Table 4—Reduced-Form Models for Change in Employment

<table>
<thead>
<tr>
<th>Independent variable</th>
<th>Model</th>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(i)</td>
<td>(ii)</td>
<td>(iii)</td>
<td>(iv)</td>
<td>(v)</td>
</tr>
<tr>
<td>1. New Jersey dummy</td>
<td>2.33</td>
<td>2.30</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>(1.19)</td>
<td>(1.20)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Initial wage gap(^a)</td>
<td>—</td>
<td>—</td>
<td>15.65</td>
<td>14.92</td>
<td>11.91</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(6.08)</td>
<td>(6.21)</td>
<td>(7.39)</td>
</tr>
<tr>
<td>3. Controls for chain and ownership(^b)</td>
<td>no</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>4. Controls for region(^c)</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>5. Standard error of regression</td>
<td>8.79</td>
<td>8.78</td>
<td>8.76</td>
<td>8.76</td>
<td>8.75</td>
</tr>
<tr>
<td>6. Probability value for controls(^d)</td>
<td>—</td>
<td>0.34</td>
<td>—</td>
<td>0.44</td>
<td>0.40</td>
</tr>
</tbody>
</table>

**Notes:** Standard errors are given in parentheses. The sample consists of 357 stores with available data on employment and starting wages in waves 1 and 2. The dependent variable in all models is change in FTE employment. The mean and standard deviation of the dependent variable are −0.237 and 8.825, respectively. All models include an unrestricted constant (not reported).

\(^a\)Proportional increase in starting wage necessary to raise starting wage to new minimum rate. For stores in Pennsylvania the wage gap is 0.

\(^b\)Three dummy variables for chain type and whether or not the store is company-owned are included.

\(^c\)Dummy variables for two regions of New Jersey and two regions of eastern Pennsylvania are included.

\(^d\)Probability value of joint \(F\) test for exclusion of all control variables.
Card and Krieger (1994) dataset (fastfood.dta) can be obtained here: http://www.stat.ucla.edu/projects/datasets/